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Fertility and Its Measurement

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Section A

Indicators of Fertility Based on Vital Statistics

Definitions

- *Fecundity*—Physiological capacity to conceive
- *Infecundity (sterility)*—Lack of the capacity to conceive
  - *Primary sterility*—Never able to produce a child
  - *Secondary sterility*—Sterility after one or more children have been born

Continued

Definitions

- *Fecundability*—Probability that a woman will conceive during a menstrual cycle
- *Fertility (natality)*—Manifestation of fecundity
- *Infertility*—Inability to bear a live birth
- *Natural fertility*—Fertility in the absence of deliberate parity-specific control

Continued

Definitions

- *Reproductivity*—Extent to which a group is replacing its own numbers by natural processes
- *Gravidity*—Number of pregnancies a woman has had
- *Parity*—Number of children born alive to a woman

Continued

Definitions

- *Birth interval*—Time between successive live births
- *Pregnancy interval*—Time between successive pregnancies of a woman

Crude Birth Rate (CBR)

- Let B = Number of births
- Let P = Mid-year population
- Let $W_{15\text{-}44}$ = Number of women of reproductive ages

Continued

Crude Birth Rate (CBR)

- *Crude Birth Rate*—Number of births per 1,000 population

$$= \frac{B}{P} * 1000$$

$$= \frac{B}{W_{15-44}} * \frac{W_{15-44}}{P} * 1000$$

Exercise

Crude Birth Rate (CBR)

- Use the following data to calculate the CBR per 1,000

Island of Mauritius, 1985

Total Births: 18,247
Total female population: 491,310
Total male population: 493,900

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Crude Birth Rate (CBR)

- The correct answer is as follows:
  - **18.5 births per 1,000 population**

Island of Mauritius, 1985

Total Births: 18,247
Total female population: 491,310
Total male population: 493,900

Crude Birth Rate (CBR)

- Crude birth rates can be standardized using the direct or the indirect method
- Example: Direct (DSBR) and indirect (ISBR) standardization of the Island of Mauritius (I.M.) 1985 crude birth rate using Mali's 1987 data as standard

Direct Standardization of Birth Rate for Mauritius Island

Age group	(Study) Rates I.M. per 1000	(Standard) Population Mali	Expected number of births, I.M.
15-19	18	725719	13063
20-24	58	574357	33313
25-29	57	536226	30565
30-34	36	443702	15973
35-39	19	379184	7204
40-44	6	325824	1955
Total		2985012	102073

Total number of births, Mali:		375117
Total number of births, I.M.:		18247
CBR Mali:	48.7	
CBR I.M.	18.5	

Source: U.N. Demographic Yearbook, *1986, 1992, 1996*

Indirect Standardization of Birth Rate for Mauritius Island

Age group	(Standard) Rates Mali per 1000	(Study) Population I.M.	Expected number of births, I.M.
15-19	79	105764	8355
20-24	159	109914	17476
25-29	171	94576	16172
30-34	140	81144	11360
35-39	107	60063	6427
40-44	50	45825	2291
Total		497286	62082

Total number of births, Mali:	375117
Total number of births, I.M.:	18247
CBR Mali:	48.7
CBR I.M.	18.5

Source: U.N. Demographic Yearbook, *1986, 1992, 1996*

$$\text{DSBR}^{\text{I.M.}} = \frac{\text{Expected births in I.M.}}{\text{Actual births in Mali}} * \text{CBR}_{\text{Mali}}$$

$$= \frac{102073}{375117} * 48.7 = 13.3$$

$$\text{ISBR}^{\text{I.M.}} = \frac{\text{Observed births in I.M.}}{\text{Expected births in I.M.}} * \text{CBR}^{\text{Mali}}$$

$$= \frac{18247}{62082} * 48.7 = 14.3$$

CBR I.M. = 18.5

Source: U.N. Demographic Yearbook, *1986, 1996*

General Fertility Rate (GFR)

- *General Fertility Rate*—Number of births per 1,000 women of reproductive ages

$$= \frac{B}{W_{15-44}} * 1000$$

$$GFR \approx 4 * CBR$$

Exercise

General Fertility Rate (GFR)

- Use the following data to calculate the GFR per 1,000 women aged 15–44:

Island of Mauritius, 1985

Age Group	Women
15-19	52 013
20-24	54 307
25-29	46 990
30-34	40 211
35-39	30 401
40-44	23 496

Total births: 18 247

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, *1986*

Continued

Exercise

General Fertility Rate (GFR)

- The correct answer is:
  - **73.7 births per 1,000 women aged 15-44**

Island of Mauritius, 1985

Age Group	Women
15-19	52 013
20-24	54 307
25-29	46 990
30-34	40 211
35-39	30 401
40-44	23 496

Total births: 18 247

Age-Specific Fertility Rate (ASFR)

- Let B_a = Number of births to women of age (group) “a”

  W_a = Number of women of age (group) “a”

  n = Number of years in age group

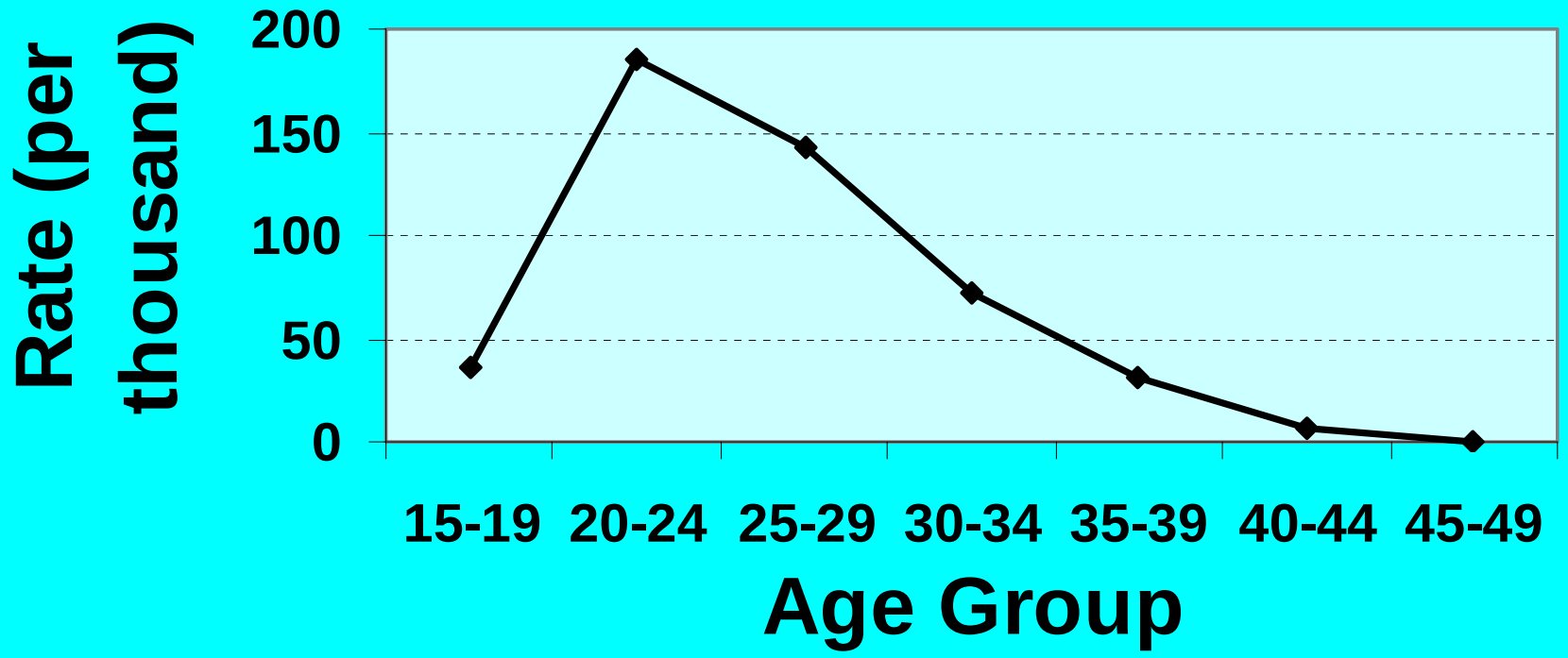
Age-Specific Fertility Rate (ASFR[a, n])

- *ASFR(a, n)*—Number of births per 1,000 women of a specific age (group)

$$= F_a = \frac{B_a}{W_a} * 1000$$

- If n = 1 , then write ASFR(a)
- Example: ASFR Poland 1984

Age-Specific Fertility Rates
Poland, 1984



Exercise

Age-Specific Fertility Rate (ASFR[a, n])

- Use the following data to calculate the ASFR per 1,000 for women age 20–24 and 25–29

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15-19	52 013	1884
20-24	54 307	6371
25-29	46 990	5362
30-34	40 211	2901
35-39	30 401	1170
40-44	23 496	268

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Age-Specific Fertility Rate (ASFR)

- The correct answers are:
  - **ASFR(20,5) = 117.3 births per 1,000 women 20–24**
  - **ASFR(25,5) = 114.1 births per 1,000 women 25–29**

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15-19	52 013	1884
20-24	54 307	6371
25-29	46 990	5362
30-34	40 211	2901
35-39	30 401	1170
40-44	23 496	268

Total Fertility Rate (TFR)

- *Total Fertility Rate*—Number of children a woman will have if she lives through all the reproductive ages and follows the age-specific fertility rates of a given time period (usually one year)

Continued

Total Fertility Rate (TFR)

- For single-year age groups

$$TFR = \sum_{a=15}^{44} \frac{B_a}{W_a} * 1000 = \sum ASFR(a) = \sum F_a$$

- For five-year age groups

$$TFR = 5 * \sum_{a=15-19}^{40-44} \frac{B_a}{W_a} * 1000 = 5 * \sum_{a=15-19}^{40-44} ASFR(a,5)$$

Continued

Total Fertility Rate (TFR)

- Example: ASFR and TFR—Poland, 1984

Age group	Ba	Wa	ASFR
15-19	43807	1230396	35.60
20-24	257872	1390077	185.51
25-29	236088	1653183	142.81
30-34	115566	1608925	71.83
35-39	38450	1241967	30.96
40-44	6627	941963	7.04
			473.74

TFR= (5 * 473.74) / 1000 = 2.4

Exercise

Total Fertility Rate (TFR)

- Use the following data to calculate the TFR per 1,000

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15–19	52,013	1,884
20–24	54,307	6,371
25–29	46,990	5,362
30–34	40,211	2,901
35–39	30,401	1,170
40–44	23,496	268

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Total Fertility Rate (TFR)

- The correct answer is as follows:
  - **TFR = 1.9 children per woman**

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15–19	52,013	1,884
20–24	54,307	6,371
25–29	46,990	5,362
30–34	40,211	2,901
35–39	30,401	1,170
40–44	23,496	268

Mean of Age of Childbearing

- For single-year groups

$$\bar{a} = \frac{\sum_{a=15}^{44} (a + 1/2) F_a}{\sum_{a=15}^{44} F_a}$$

- For five-year age groups

$$\bar{a} = \frac{\sum (a + 2.5) F_a}{\sum F_a}$$

Variance of Age of Childbearing

$$s^2 = \frac{\sum_{a=15}^{44} \left(a - \bar{a} \right)^2 F_a}{\sum_{a=15}^{44} F_a}$$

Exercise

Mean and Variance of Age of Childbearing

- Use the following data to calculate the mean and variance of age of childbearing

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15–19	52,013	1,884
20–24	54,307	6,371
25–29	46,990	5,362
30–34	40,211	2,901
35–39	30,401	1,170
40–44	23,496	268

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Mean and Variance of Age of Childbearing

- The correct answers are as follows:
  - Mean age of childbearing **= 27.9 years**
  - Variance of age of childbearing **= 38.2 years**

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15–19	52,013	1,884
20–24	54,307	6,371
25–29	46,990	5,362
30–34	40,211	2,901
35–39	30,401	1,170
40–44	23,496	268

Mean and Median Age of Mothers

- Mean:

$$\frac{\sum_{a=\alpha}^{\beta}(a+\frac{1}{2})*B_a}{\sum B_a}$$

Median x such that:

$$\frac{\sum_{a=15}^{x} B_a}{\sum_{a=15}^{44} B_a} = 0.5$$

- B_a = Number of births to women age a

Exercise

Mean and Median Age of Mothers

- Use the following data to calculate the mean and median age of mothers

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15–19	52,013	1,884
20–24	54,307	6,371
25–29	46,990	5,362
30–34	40,211	2,901
35–39	30,401	1,170
40–44	23,496	268

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Mean and Median Age of Mothers

- The correct answers are as follows:
  - **Mean age of mothers = 26.9 years**
  - **Median age of mothers = 24.7 years**

Island of Mauritius, 1985

Age Group of Mother	Women	Births
15Ğ19	52,013	1,884
20Ğ24	54,307	6,371
25Ğ29	46,990	5,362
30Ğ34	40,211	2,901
35Ğ39	30,401	1,170
40Ğ44	23,496	268

Parity

- Mean

$$\sum_{i=1}^{I=\max(i)} M_i$$

Median x such that:

$$\sum_{i=1}^{x} m_i = 0.5$$

- M_i = Proportion of women at or above parity i
- m_i = Proportion of women at parity i

Marital Fertility Rate (MFR)

- Let B_m = Number of marital births

  B_u = Number of non-marital births

  $W^m_{15–44}$ = Number of married women of age 15–44

  $W^u_{15–44}$ = Number of unmarried women of age 15–44

General Marital Fertility Rate (GMFR)

- *General Marital Fertility Rate*—Number of births per 1,000 married women of reproductive ages

$$= \frac{B}{W^{m}_{15\text{-}44}} * 1000$$

Marital Fertility Rate (MFR)

- *Marital Fertility Rate*—Number of marital births per 1,000 married women of reproductive ages

$$= \frac{B_m}{W^m_{15\text{-}44}} * 1000$$

"Out-of-Wedlock" (Non-Marital) Fertility Rate

- *"Out-of-Wedlock" (Non-Marital) Fertility Rate*—Number of non-marital births per 1,000 unmarried women of reproductive ages

$$= \frac{B_u}{W^u_{15\text{-}44}} * 1000$$

Some Relationships

$$\frac{B_m}{W^m_{15\text{-}44}} + \frac{B_u}{W^u_{15\text{-}44}} \neq \frac{B}{W_{15\text{-}44}}$$

- But

$$\frac{B_m}{W^m_{15\text{-}44}} * \frac{W^m_{15\text{-}44}}{W_{15\text{-}44}} + \frac{B_u}{W^u_{15\text{-}44}} * \frac{W^u_{15\text{-}44}}{W_{15\text{-}44}} = \frac{B}{W_{15\text{-}44}}$$

Age-Specific Marital Fertility Rate (ASMFR)

- Let B^m_a = Number of marital births to women of age group “a”

  W^m_a = Number of married women in age group “a”

  $B^m_{(d)}$ = Number of marital births to women married for “d” years

  $W^m_{(d)}$ = Number of women married for “d” years

Continued

Age-Specific Marital Fertility Rate (ASMFR)

- *Age-Specific Marital Fertility Rate*—Number of marital births per 1,000 married women of age (group) “a”

$$= \frac{B_{mla}}{W_a^m} * 1000$$

Duration (of Marriage)—Specific Fertility Rate (DSFR)

- *DSFR*—Number of marital births per 1,000 women who have been married for duration “d”

$$= \frac{B_{l(d)}}{W^{m}(d)} * 1000$$

Order-Specific Fertility Rate (OSFR)

- Let B^i = Number of births of order "i", i>0

  B^i_a = Number of order "i" births to women in age group "a"

  W_a = Number of women in age group "a"

  $W_{15\text{-}44}$ = Number of women of age 15–44 (or 15–49)

Continued

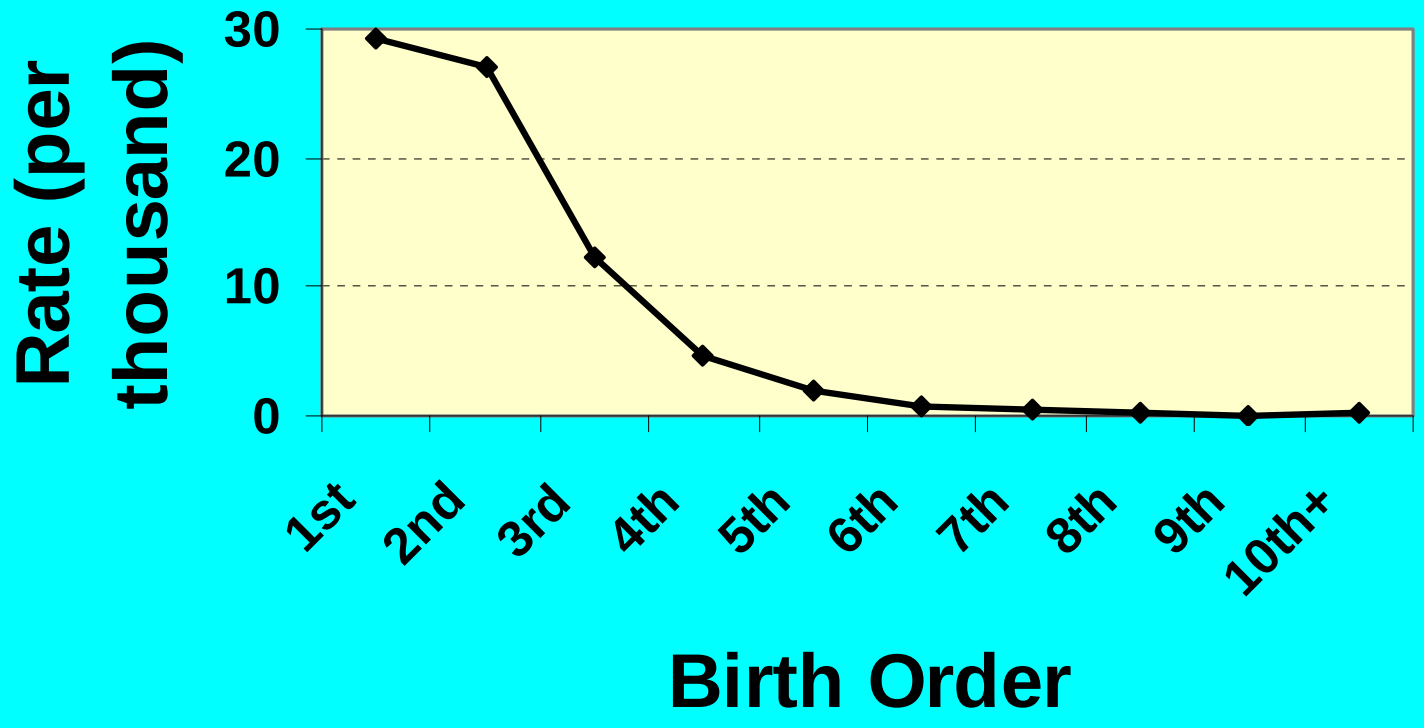
Order-Specific Fertility Rate (OSFR)

- *Order-Specific Fertility Rate*—Number of order "i" births per 1,000 women of reproductive ages

$$= \frac{B^{i}}{W_{15-44}} * 1000$$

- Example: OSFR Poland 1984

Order-Specific Fertility Rates
Poland, 1984



Exercise

Order-Specific Fertility Rate (OSFR)

- Use the following data to calculate the OSFR for birth orders 1 and 3

Island of Mauritius, 1985

Age Group of Mother	Women	Number of Birth Order 1	3
15–19	52,013	1,521	40
20–24	54,307	3,317	678
25–29	46,990	1,638	1,132
30–34	40,211	496	665
35–39	30,401	142	215
40–44	23,496	24	30

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Order-Specific Fertility Rate (OSFR)

- The correct answers are as follows:
  - **OSFR(1) = 28.8 births of order 1 per 1,000 women 15–44**
  - **OSFR(3) = 11.2 births of order 3 per 1,000 women 15–44**

Island of Mauritius, 1985

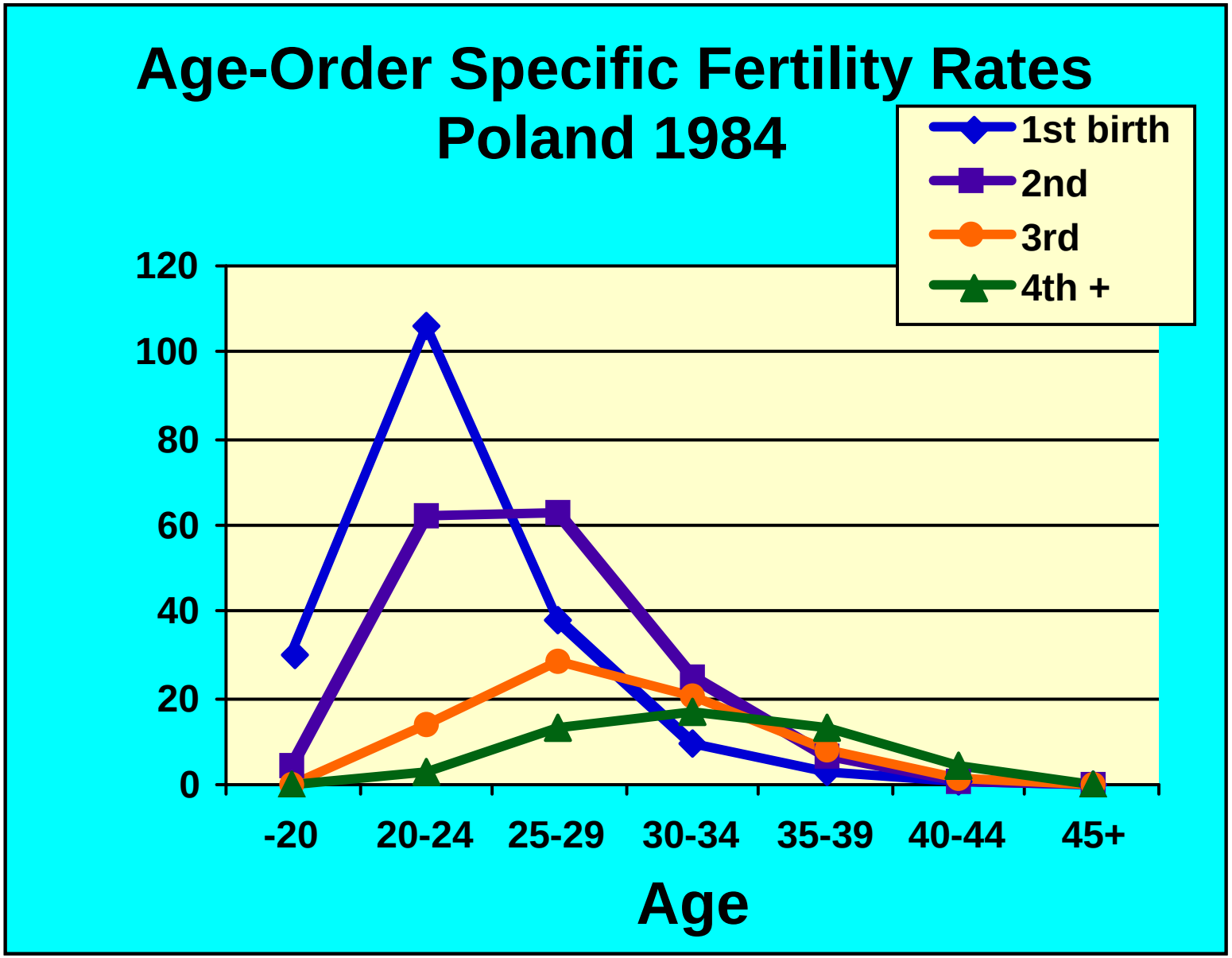
Age Group of Mother	Women	Number of Birth Order 1	3
15–19	52,013	1,521	40
20–24	54,307	3,317	678
25–29	46,990	1,638	1,132
30–34	40,211	496	665
35–39	30,401	142	215
40–44	23,496	24	30

Age-Order Specific Fertility Rate (AOSFR [a,I])

- *AOSFR(a,i)*—Number of order "i" births per 1,000 women of age (group) "a"

$$= \frac{B_a^i}{W_a} * 1000$$

- Example: AOSFR Poland 1984


Age-Order Specific Fertility Rates
Poland 1984
1st birth
2nd
3rd
4th +
120
100
80
60
40
20
0
-20
20-24
25-29
30-34
35-39
40-44
45+
Age

Exercise

Age-Order Specific Fertility Rate (AOSFR)

- Use the following data to calculate the AOSFR for birth order 3 in age group 25–29

Island of Mauritius, 1985

Age Group of Mother	Women	Birth order 1	3
15Ğ19	52,013	1,521	40
20Ğ24	54,307	3,317	678
25Ğ29	46,990	1,638	1,132
30Ğ34	40,211	496	665
35Ğ39	30,401	142	215
40Ğ44	23,496	24	30

Source: U.N. Demographic Yearbook, *1986*

Exercise Answer

Age-Order Specific Fertility Rate (AOSFR)

- The correct answer is as follows:
  - **AOSFR = 24.1 births of order 3 per 1,000 women 25–29**

Island of Mauritius, 1985

Age Group of Mother	Women	Birth order 1	3
15–19	52,013	1,521	40
20–24	54,307	3,317	678
25–29	46,990	1,638	1,132
30–34	40,211	496	665
35–39	30,401	142	215
40–44	23,496	24	30

Age-Order Specific Fertility Rate (AOSFR[a,i])

- Note:

$$\sum_{i=1}^{\infty} AOSFR\ (a,i) = ASFR\,(a)$$

$$\sum_{a=15}^{44} AOSFR\ (a,i) * \frac{W_a}{W_{15\text{-}44}} = OSFR(i)$$

Cumulative Order Specific Birth Rate (COSFR[i,a])

- Cumulative up to age “a”
- *COSFR(i,a)*—Total number of order “i” births per 1,000 women of age (group) less than or equal to “a”

$$= \sum_{x=0}^{a} AOSFR(i,x)$$

Birth Probability

- Let B^i = Number of births of order “i”, i>0, in year “t”

  W^{i-1} = Number of women of parity “i-1” at beginning of year “t”

Birth Probability, BP(i)

- *Birth Probability*—Probability of having an "ith" order birth in a given year for women who already have "i-1" births

$$= \frac{B^{i}}{W^{i-1}}$$

Birth Probability

- May be age-specific as well
- Birth probabilities are the most sensitive indicators of temporal change in the pace of childbearing

Paternal Fertility Rate

- Let B = Number of births

  $M_{15\text{-}54}$ = Number of men of age 15–54

General Fertility Rate of Men (GFR_m)

- *General Fertility Rate of Men*—Number of births per 1,000 men age 15 to 54

$$= \frac{B}{M_{15-54}} * 1000$$

Summary

- Fertility data are collected from vital statistics, censuses, or surveys
- Vital statistics principally provide birth statistics
- Many indicators have been developed to understand and explain the fertility patterns in populations based on vital statistics

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Section B

Indicators of Reproduction Based on Vital Statistics

Gross Reproduction Rate (GRR)

- Let B^f = Number of female births

  B^{m+f} = Number of male and female births, i.e., all births

Continued

Gross Reproduction Rate (GRR)

- *Gross Reproduction Rate*—Number of daughters expected to be born alive to a hypothetical cohort of women (usually 1,000) if no one dies during childbearing years and if the same schedule of age-specific rates is applied throughout the childbearing years

Continued

Gross Reproduction Rate (GRR)

$$GRR = \sum ASFR(a) * \frac{B^{f}(a)}{B^{m+f}(a)}$$

$$GRR = TFR * (\text{Proportion of female births})$$

- If the sex ratio at birth is assumed constant across ages of women

Exercise

Gross Reproduction Rate (GRR)

- Use the following data to calculate the GRR

United States, 1990

Age Group of Mother	Women	Births Total	Births Males
15-19	8 651	522	267
20-24	9 345	094	560
25-29	10 617	1277	653
30-34	10 986	886	454
35-39	10 061	318	163
40-44	8 924	49	25

Numbers are in 1,000s

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Sources: U.S. Census *1990 and* Vital Statistics of the U.S. Vol. I

Exercise Answer

Gross Reproduction Rate (GRR)

- The correct answer for the gross reproduction rate is as follows:
  - **1.01 daughters per woman**

United States, 1990

Age Group of Mother	Women	Births	
		Total	Males
15-19	8 651	522	267
20-24	9 345	094	560
25-29	10 617	1277	653
30-34	10 986	886	454
35-39	10 061	318	163
40-44	8 924	49	25

Numbers are in 1,000s

Net Reproduction Rate (NRR)

- Let L^f_a = Life table person-years lived by women in age group "a"

  ℓ_0 = Radix of life table

  B^f = Number of female births

  B^{m+f} = Number of male and female births

Continued

Net Reproduction Rate (NRR)

- *Net Reproduction Rate*—Average number of daughters expected to be born alive to a hypothetical cohort of women if the same schedule of age-specific fertility and mortality rates applied throughout the childbearing years

Continued

Net Reproduction Rate (NRR)

- For single-year age groups

$$NRR = \sum ASFR * \frac{{}_1L_a^f}{\ell_0^f} * \frac{B^f}{B^{m+f}}$$

- For five-year age groups

$$NRR = \sum ASFR * \frac{{}_5L_a^f}{\ell_0^f} * \frac{B^f}{B^{m+f}}$$

Exercise

Net Reproduction Rate (NRR)

- Use the following data to calculate the NRR

United States, 1990

Age Group of Mother	Women	Births	
		Total	Males
15-19	8 651	522	267
20-24	9 345	094	560
25-29	10 617	1277	653
30-34	10 986	886	454
35-39	10 061	318	163
40-44	8 924	49	25

Numbers are in 1,000s

Sources: U.S. Census *1990 and* Vital Statistics of the U.S. Vol. I

Exercise Answer

Net Reproduction Rate (NRR)

- The correct answer for the NRR is as follows:
  - **1.00 daughter per woman**

United States, 1990

Age Group of Mother	Women	Births		Stationary Population
		Total	Males	$_5L_x$
15-19	8 651	522	267	493 629
20-24	9 345	094	560	492 399
25-29	10 617	1277	653	490 989
30-34	10 986	886	454	489 203
35-39	10 061	318	163	486 812
40-44	8 924	49	25	483 465

Numbers are in 1,000s

Net Reproduction Rate (NRR)

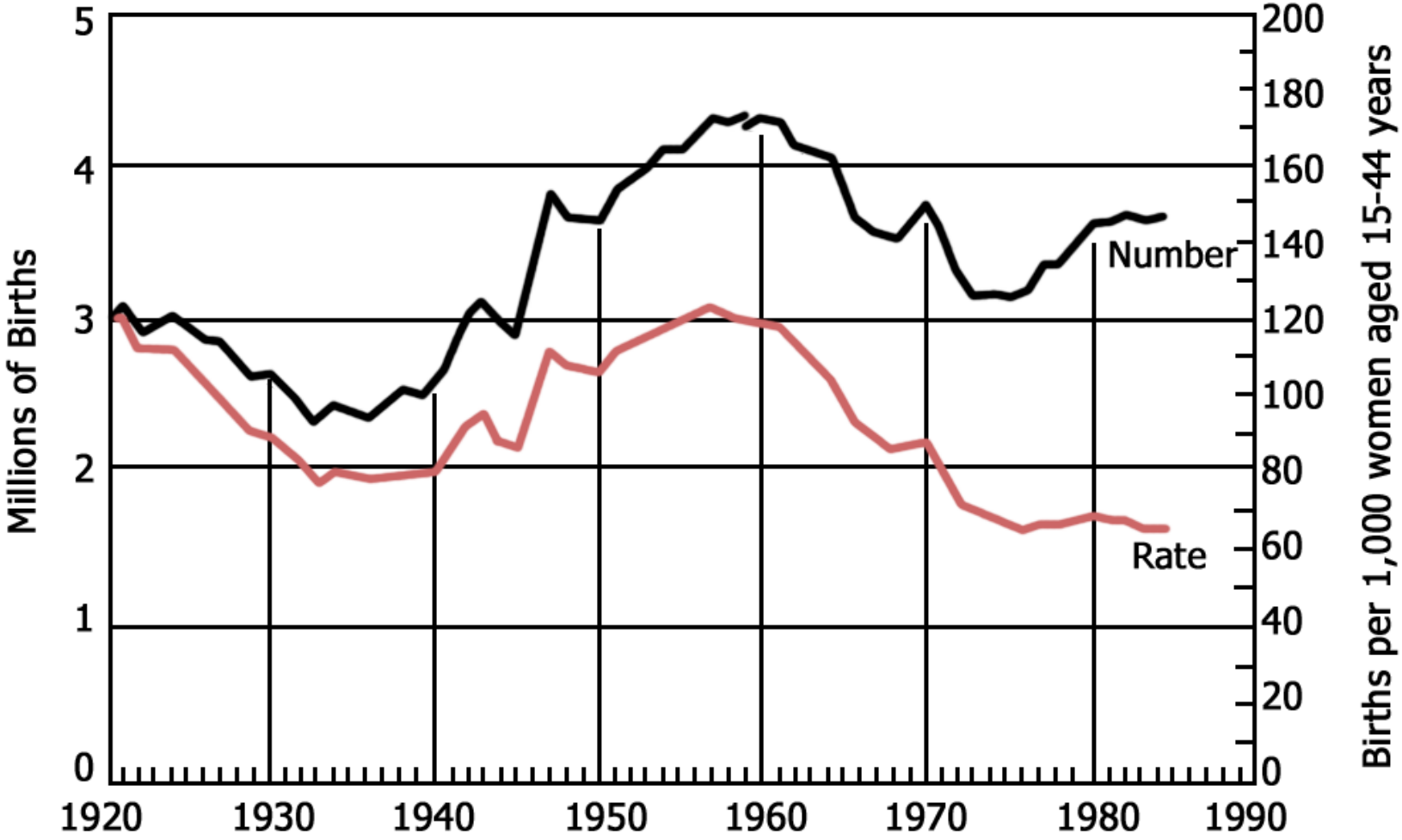
$$NRR \approx \ell(\bar{a}) * GRR$$

- Where $\ell(\bar{a})$ = Life table probability of surviving beyond $\bar{a}$

  $\bar{a}$ = mean age of childbearing

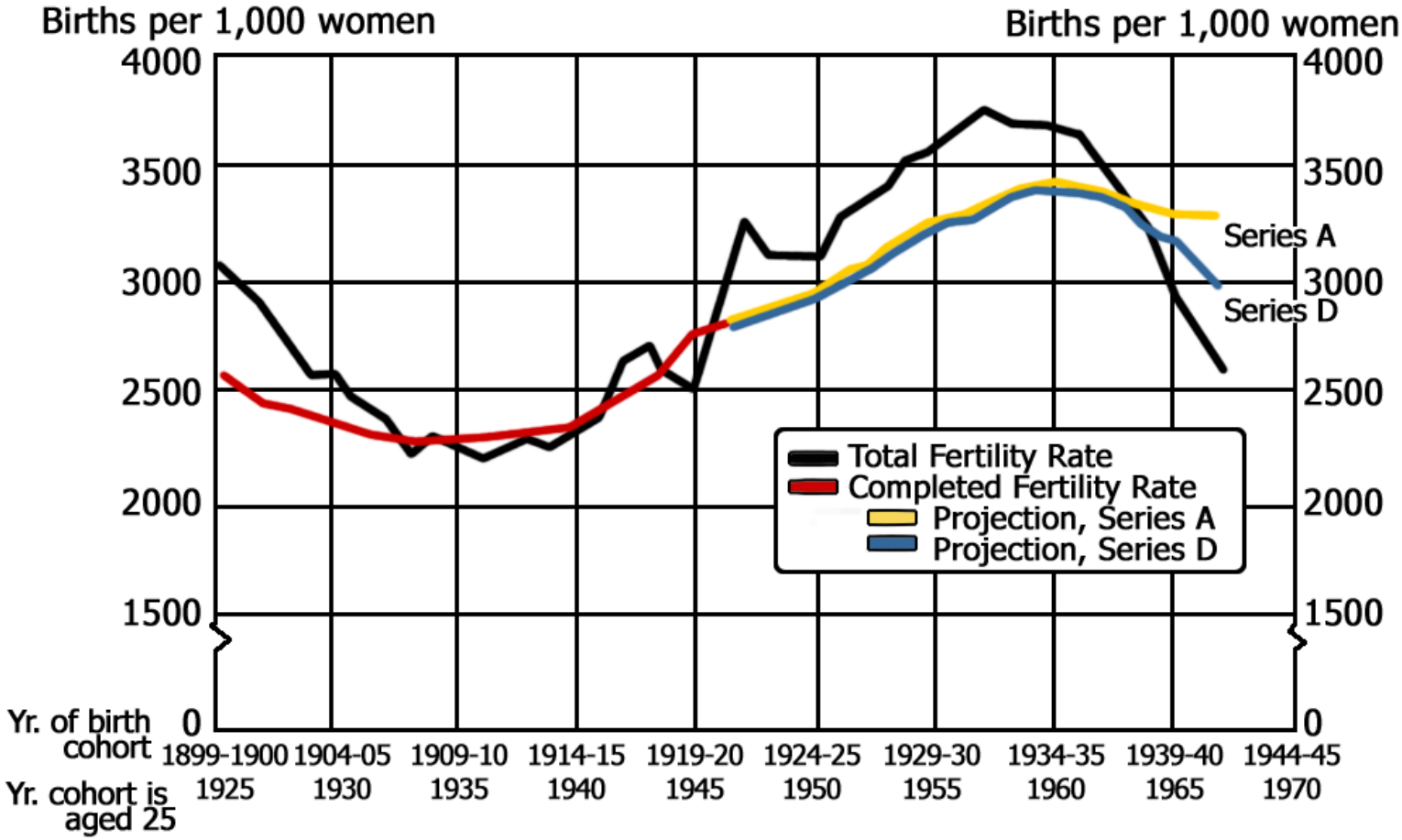
  usually $20 < \bar{a} < 30$

Live Births and Fertility Rates: United States, 1920–1988



Source: U.S. Department of Health and Human Services, Monthly Vital Statistics Report, *Vol. 39, No. 4, Supplement, August 15, 1990*

Completed Fertility Rates for Birth Cohorts of Women



Source: U.S. Bureau of the Census, Current Population Reports, *Series P-25, No. 381, tables A-1 and A-2*

Reproductivity

- Reproductivity is usually studied in terms of mothers and daughters because of the following reasons:
  - The fecund period for females is shorter than it is for males
  - Characteristics such as age are much more likely to be known for the mothers of illegitimate babies than for their fathers

Summary

- Reproductivity data are collected from vital statistics
- Many indicators have been developed to understand and explain reproductivity patterns in populations

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Section C

Indicators of Fertility Based on Censuses and Surveys

Child-Woman Ratio (CWR)

- *Child-Woman Ratio*—Number of children zero to four years old relative to number of women of reproductive ages

Continued

Child-Woman Ratio (CWR)

$$CWR = \frac{P_{0-4}}{W_{15-44}} * 1000$$

- Where $P_{0\text{-}4}$ = Mid-year population of persons age 0-4 years

  $W_{15\text{-}44}$ = Number of women of reproductive ages

Exercise

Child-Woman Ratio (CWR)

- Use the following data to calculate the CWR

Household population composition by age and sex, Kenya 1998

Age group	Male	Female	Total
0-4	268873	256705	525578
…			
15-19	189272	192067	381340
20-24	130899	158825	289723
25-29	116747	142204	258951
30-34	100827	99727	200555
35-39	88445	103421	191866
40-44	67218	68332	135550

Source: Demographic and Health Survey, *Kenya 1998*

Exercise Answer

Child-Woman Ratio (CWR)

- The correct answer is as follows:
  - **687.4 children 0-4 per 1,000 women 15-44**

Household population composition by age and sex, Kenya 1998

Age group	Male	Female	Total
0-4	268873	256705	525578
…			
15-19	189272	192067	381340
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Children Ever Born (CEB)

- *Children Ever Born*—Total number of children a woman has ever given birth to
- The survival status of the children is not considered here
- Almost all censuses tabulate mean CEB by marital status and by age of the mother

Parity Progression Ratios (PPR[i])

- Let N = Random variable for number of births

  m_i = Proportion of women of parity "i"

  M_i = Proportion of women at or above parity "i"

Continued

Parity Progression Ratios (PPR[i])

- *Parity Progression Ratios*—Probability that a woman has an (i+1st) birth given that she already has had "i" births

$$= a_i$$

$$= \frac{M_{i+1}}{M_i}$$

$$= \text{Prob}\,(N \geq i+1 \mid N \geq i)$$

Exercise

Parity Progression Ratios (PPR)

- Given the following percentages of women at parity “i” for women 45-49 in 1995 in Colombia, calculate PPR(2)

Parity	Percentage
0	9.2
1	7.5
2	13.9
3	18.6
4	15.6
5+	35.2

Source: Demographic and Health Survey, *Colombia 1995*

Exercise Answer

Parity Progression Ratios (PPR)

- The correct answer for the PPR(2) is as follows:
  - **0.83** (i.e., there is an 83% chance that a woman 45–49 has a second birth given she already has had a first birth)

Parity	Percentage
0	9.2
1	7.5
2	13.9
3	18.6
4	15.6
5+	35.2

Parity Progression Ratios (PPR)

- Note: a_0 = Prob (ever give birth)
  $1-a_0$ = Prob (never have a birth)
  $M_0 = 1$
- a_i need not decrease with increasing I
- M_i does decrease

Proportion of Women of Parity i(mi)

- Is an unconditional probability

$$= M_i - M_{i+1}$$

$$= \text{Prob}(N \geq i) - \text{Prob}(N \geq i+1)$$

$$= a_0 * a_1 * \ldots * a_{i-1} * (1 - a_i)$$

Continued

Proportion of Women of Parity i(mi)

- Given the following percentages of women at parity “i” for women 45–49 in 1995 in Colombia, verify the relationships indicated on the previous slide

Parity	Percentage
0	9.2
1	7.5
2	13.9
3	18.6
4	15.6
5+	35.2

Source: Demographic and Health Survey, *Colombia 1995*

Mean Parity

$$= \sum_{i=1}^{\max(i)} M_i = \sum_{i=1}^{\max(i)} i * m_i$$

Exercise

Mean Parity

- Given the following percentages of women at parity "i" for women 45–49 in 1995 in Colombia, calculate the mean parity using both formulas on the previous slide (assume that women at parity 10+ are on average at parity 11)

Parity	Percentage	Parity	Percentage
0	9.2	6	8.0
1	7.5	7	5.9
2	13.9	8	3.2
3	18.6	9	3.2
4	15.6	10+	4.8
5	10.1		

Source: Demographic and Health Survey, *Colombia 1995*

Exercise Answer

Mean Parity

- The correct answer is as follows:
  - **Mean parity = 4.01**

Parity	Percentage	Parity	Percentage
0	9.2	6	8.0
1	7.5	7	5.9
2	13.9	8	3.2
3	18.6	9	3.2
4	15.6	10+	4.8
5	10.1		

Estimation of GFR from Census Data on Ratio of Children to Women

- Let $P_{0\text{-}4}$ = Enumerated number of children under 5

  $W_{15\text{-}44}$ = Enumerated number of women of age 15–44

  $W_{20\text{-}49}$ = Enumerated number of women of age 20–49

Continued

Estimation of GFR from Census Data on Ratio of Children to Women

- Also let ℓ_0 = Radix of life table

  ${}_nL_a$ = Life table person-years lived between the ages “a” and “a+n” (female life table)

Continued

Estimation of GFR from Census Data on Ratio of Children to Women

$$GFR_{est} = \frac{P_{0\text{-}4} * \frac{\ell_0}{{}_5L_0}}{\frac{W_{15-44} + W_{20-49}}{2} * \frac{{}_{30}L_{15}}{\frac{{}_{30}L_{20} + {}_{30}L_{15}}{2}}}$$

Continued

Estimation of GFR from Census Data on Ratio of Children to Women

- Note:
  - Assumes that a life table is available
  - Estimated GFR may be used in the absence of birth statistics to compare fertility levels in various areas

Continued

Estimation of GFR from Census Data on Ratio of Children to Women

- The life table survivorship functions are inverted in order to estimate the number of persons at the mid-point of the preceding five-year period
- A lexis diagram makes it easier to understand this calculation

Summary

- Fertility data are collected from vital statistics, censuses, or surveys

Continued

Summary

- Censuses provide the following:
  - Data on births and fertility
  - Statistics on children by family status of the parents
  - Population data on fertility-related variables
  - Population bases for calculating various types of fertility measures

Continued

Summary

- Surveys provide the following:
  - Same type of data as censuses
  - Additional detailed data on special aspects of fertility, including number and timing of births, marriages, pregnancies, birth intervals, and birth interval components

Continued

Summary

- Many indicators have been developed to understand and explain the fertility patterns in populations

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Section D

Relationship among Indicators, Indicators and Models of Birth Intervals, and Fertility Models

Relations among Indicators

- At age 50:

$$\sum_{i=1}^{\max(i)} \text{COSFR}(i,50) = \text{Completed cohort fertility rate} = \text{Mean CEB} = \sum_{i=1}^{\max(i)} M_i$$

Continued

Relations among Indicators

$$\sum_{i=1}^{\max(i)} \mathrm{OSFR}\ (i) = \sum_{i=1}^{\max\ (i)} \frac{B_i}{W_{15-44}}$$

$$= \frac{B}{W_{15-44}}$$

$$= \mathrm{GFR}$$

Pregnancy Histories and Birth Histories

- Data needed
  - Age or birth date of woman
  - Dates of pregnancy terminations
  - Type of termination (live birth or not)

Continued

Pregnancy Histories and Birth Histories

- Two ways of collecting data
  - Forward, i.e., from first to last birth
  - Backward, i.e., from last to first birth

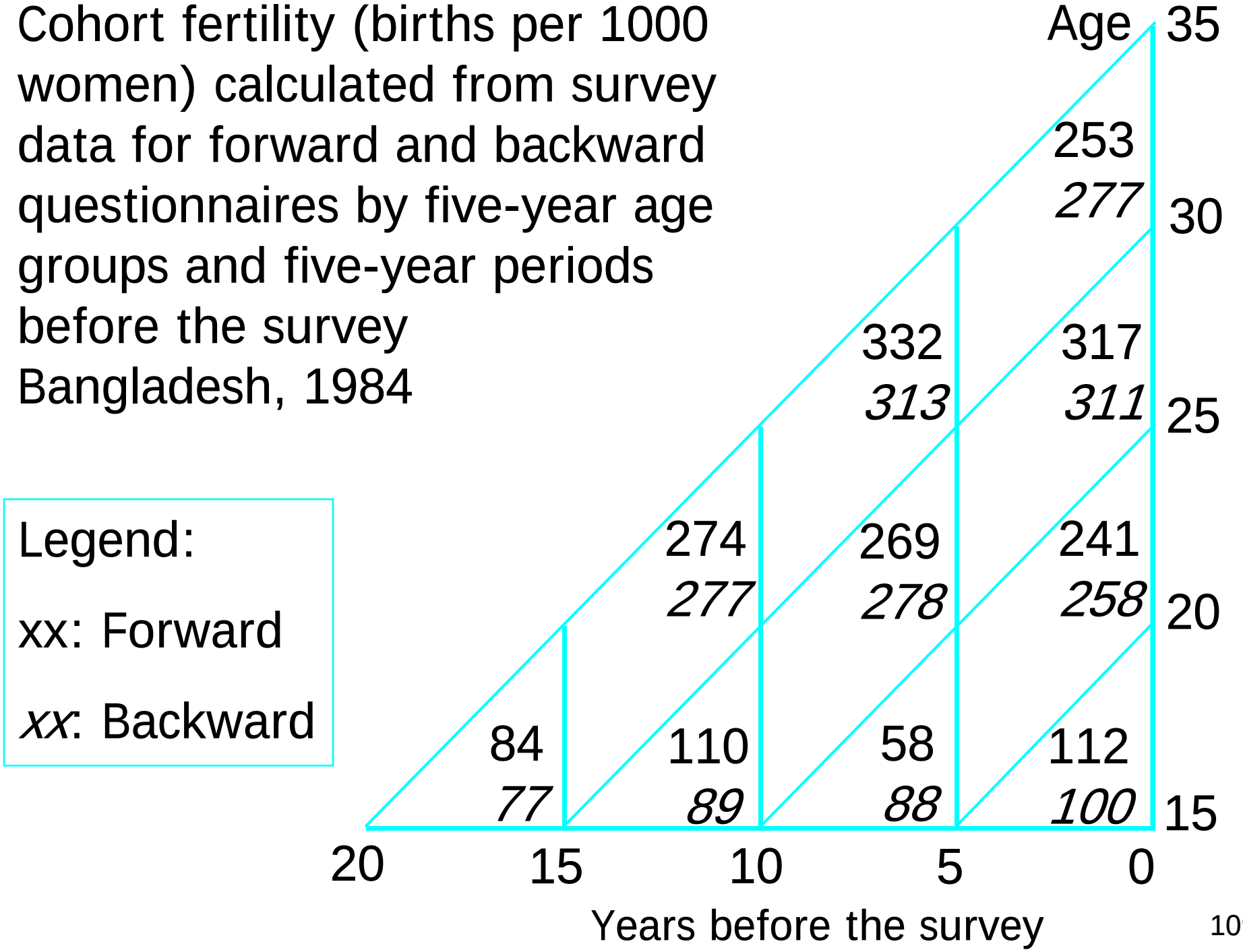
Continued

Pregnancy Histories and Birth Histories

- Problems
  - Dating of events
  - Forgetting events
  - Age misreporting

Continued

Cohort fertility (births per 1000 women) calculated from survey data for forward and backward questionnaires by five-year age groups and five-year periods before the survey
Bangladesh, 1984



Birth Interval Measures and Models

- Let NS = Time from previous pregnancy outcome to resumption of ovulation (postpartum non-susceptibility subinterval)
- For first order births use dates of marriage or beginning of sexual relations

Continued

Birth Interval Measures and Models

- Also let C = Time from resumption of ovulation to next conception (conception-wait subinterval)

  G = Time from conception to next pregnancy outcome (gestational subinterval)

  W = Waiting time due to non-live births

Pregnancy and Live Birth Intervals

- Pregnancy interval
  - PI = Interval between two pregnancies

$$= NS + C + G$$

- Live birth interval
  - LBI = Interval between two live births

$$= PI + W$$

Observed Birth Intervals

- Closed intervals
- Left censored intervals
- Right censored intervals
- Life table methods can be used to incorporate the different intervals and calculate median birth interval lengths

Birth Interval

- Note:
  - Mean birth interval for women is different from mean birth interval for births in a given time period
  - The latter is shorter

Birth Interval

Mathematical relationships

- In populations with nothing changing

  Let p = Fecundability (assumed fixed)

  e = Effectiveness of contraception

  u = Proportion using contraception

  s = Non-susceptible period (constant)

Mean Birth Interval

$$\text{Mean birth interval} = \frac{1}{(\,p\,(1 - ue)\,)} + s$$

$$\frac{1}{\text{Mean Birth Interval}} = \text{Fertility rate for fecund women}$$

Mean Conception-Wait Subinterval (MC)

- *Mean Conception-Wait Subinterval*—Mean time it takes to get pregnant under natural fertility
- Constant fecundability

$$MC = 1 * p + 2(1 - p)p + 3(1 - p)^2 p + \ldots$$

$$= \frac{1}{p}$$

p = Monthly probability of conception (assumed fixed in time and for all women)

Heterogeneous Fecundability

- Probability of conception in month “k”
- “p” is assumed to vary between women with distribution “f(p)”

$$= \int_0^1 p\, q^{k-1} f(p)\, dp$$

  - Where q = 1-p

Probability of Conceiving

In Month "k" Given No Conception Before Then

$$= 1 - \frac{\int_0^1 q^k \, f(1-q)\, dq}{\int_0^1 q^{k-1} \, f(1-q)\, dq}$$

- Note
  - Here the probability of conception decreases over time
  - This has important implications for the study of infertility

Fertility Models

Coale and Trussell

- Let r(a) = Observed fertility schedule at age “a”
  n(a) = Natural fertility pattern at age “a”
  M = Level of fertility (estimated)
  m = Degree to which fertility control is practiced (estimated)
  v(a) = Estimated from U.N. Demographic Yearbook 1965; data for 43 fertility schedules

Continued

Fertility Models

Coale and Trussell

$$\frac{r(a)}{n(a)} = M * e^{(m * v(a))}$$

Fertility Models

Bongaarts

- Bongaarts model of intermediate fertility variables

  Let C_m = Index of marriage

  C_c = Index of contraception

  C_a = Index of induced abortion

  C_i = Index of post-partum infecundability

  K = Estimated total natural fertility rate (15.3 [Bongaarts 1982])

- Each “C” varies between 0.0 and 1.0

Continued

Fertility Models
Bongaarts

- Methods of estimation of each “C” are provided by Bongaarts (1982)

$$TFR = C_m * C_c * C_a * C_i * K$$

Summary

- Information on women's fertility can be obtained by asking them to report their pregnancy and birth histories
- Indicators and models have been developed to understand and explain the fertility patterns in populations