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JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

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Summary Measures (Ratio, Proportion, Rate)

Marie Diener-West, PhD
Johns Hopkins University

Summary Measures

- Public health questions are about **populations**
- Information about population characteristics is often summarized in an **index**
- Changes in population characteristics can be assessed by comparing summary measures

Indices Used to Summarize Information

- A **ratio** can be written as one number divided by another (a fraction) of the form **a/b**
  - Both a and b refer to the frequency of some event or occurrence
- A **proportion** is a ratio in which the numerator is a subset (or part) of the denominator and can be written as **a/(a+b)**
  - A relative frequency
- A **rate** is a ratio of the form **a*/ (a+b)**
  - a* = the frequency of events during a certain time period
  - a+b = the number at risk of the event during that time period
- A rate may or may not be a proportion

Properties of Ratios

- R = a/b
- Often a ratio R is rescaled by multiplying by a constant k
  - Where k is a number such as 10, 100, 1,000, or 10,000
- R is always > 0
- R may or may not have units

Examples of Ratios: Example 1

- $R = \frac{\text{observed number of AIDS cases in County A during June}}{\text{expected number of AIDS cases in County A during June}}$

- Example: 40 cases / 20 cases = 2
- No units

Examples of Ratios: Example 2

- R = number of hospitals / (population size)
- R may be multiplied by k = 10,000
- Units = hospitals per 10,000 people
- Suppose
  - R = 4 hospitals/20,000 people
    = 0.0002 hospitals per person
  - R*k = 0.0002 * 10,000
    = 2 hospitals per 10,000 people
  - Units = hospitals per 10,000 people

Examples of Ratios: Odds

- p = proportion of people with disease
- 1–p = proportion of people without disease
- O = p / (1–p) = “odds” of disease
- No units

Examples of Ratios: Odds Ratio

- OR = odds ratio
- $$OR = \frac{\text{odds of disease in Population 1}}{\text{odds of disease in Population 2}}$$
- $$OR = \frac{O_1}{O_2}$$
- No units

Examples of Ratios: Standardized Mortality Ratio

- SMR = standardized mortality ratio
- SMR = the ratio of the number of events observed in the study population to the number that would be expected if the study population were exposed to the same specific rates as the standard population
- SMR = O/E
- No units

Properties of Proportions

- n = the number of individuals in a population
- x = the number of individuals in the same population possess characteristic C
- p = proportion in the population with characteristic C is equal to x/n

Properties of Proportions

- p takes on values between 0 and 1 (p is a fraction)
- p has no units
- p may be multiplied by a constant k
  - Where k is a number such as 100, 1,000, or 100,000

Example of Proportion

- Proportionate mortality
- In 1995, 53% of all deaths in Africa were children under age 5
- p = 0.53 = 53% = 53 per 100 = 530 per 1,000

Ratios, Proportions, and Rates

- A proportion is always a ratio
- A rate is always a ratio
- A rate may or may not be a proportion

Properties of Rates

- The **calendar time period** is the same in both the numerator and denominator of a rate
- A rate expresses the relative frequency of an event per unit time (“risk”)

Examples of Rates in Vital Statistics

- **Infant mortality rate (IMR)** = number of infant deaths per 1,000 live births during a calendar year
  - The IMR is a ratio
  - The IMR is **not** a proportion because the numerator is not necessarily part of the denominator (some infants may have been born during the previous calendar year)
- **Fertility rate** = number of live births per 1,000 women aged 15–44 during a calendar year
  - The fertility rate is both a ratio and a proportion

Examples of Rates in Vital Statistics

- **Annual crude death rate =**

$$\frac{\text{total \# deaths in a calendar year}}{\text{total midyear population}}$$

- **Annual age-specific death rate for ages 1–4 =**

$$\frac{\text{total \# deaths aged } 1-4 \text{ in a calendar year}}{\text{midyear population aged } 1-4}$$

Examples of Rates in Vital Statistics

- **Percent of all deaths which are ages 1–4 =**

$$\frac{\text{total \# deaths aged } 1-4 \text{ in calendar year}}{\text{total \# deaths in calendar year}} \text{ x } 100$$

- **Percent of all deaths ages 1–4 due to malignancy =**

$$\frac{\text{\# cancer}-\text{related deaths aged } 1-4 \text{ in calendar year}}{\text{total \# deaths in calendar year}} \text{ x } 100$$

Examples of Rates: Incidence and Prevalence

- **Incidence rate =**

$$\frac{\#\ \text{new cases of specific disease in calendar year}}{\text{total midyear population}}$$

- **Prevalence rate (point prevalence) =**

$$\frac{\#\ \text{cases [old or new] of specific disease at time t}}{\text{total population at time t}}$$

- **Prevalence rate (period prevalence) =**

$$\frac{\#\ \text{cases diagnosed with a specific disease in a time period}}{\text{total population in the time period}}$$

Person-Time and Rates

- Individuals may be exposed to the risk of an event for varying amounts of time during a total time period of a certain length due to:
  - Entering the time period later
  - Leaving the time period earlier
  - Experiencing the event of interest
- **Person-time**
  - Is a calculation combining persons and time
  - Is the sum of the individual units of time that people have been exposed to the risk of an event
  - Is used in the denominator of person-time rates
  - Is often used in epidemiology and vital statistics

Definitions Useful in Person-Time Analysis

- T = length of the time period of interest
- N(T) = number of people exposed to risk of the event during T
- E(T) = sum of the time units that each person is exposed to risk of the event (total person-time)
- D(T) = number of people with the event during T
- $R = \dfrac{D(T)}{E(T)}$
- $R = \dfrac{\text{number of events}}{\text{total person-time of exposure}}$

Example 1: Person-Years

- Suppose during a two-year period of time, **10 episodes of diarrhea** at a day-care center were reported
- Thirty-five children attend the day-care center, for varying fractions of the two-year period, for a total of **50 child-years**

- $R = \dfrac{10 \text{ diarrhea episodes}}{50 \text{ child} - \text{years of observation}}$

  = 0.20 episodes per child-year

Approximation of Person-Time in Vital Statistics

- In vital statistics, the exact exposure times rarely are known
- E(T), the denominator, may be approximated by multiplying the mid-period population, N, by the length of the time-period, T
- Then, $R = \dfrac{D(T)}{N * T}$

Example 2: Person-Years

- Suppose during a two-year period of time, **10 episodes of diarrhea** at a day-care center were reported
- Suppose **30 children** were enrolled in the day-care center at the **mid-period of one year**
- $R = \dfrac{\text{(10 diarrhea episodes)}}{\text{(30 children attending the daycare center for 2 years}}$

  $= 10/60$

  = **0.17 episodes per child-year**

Assessing Change in Two Rates

- **Absolute (arithmetic) change =** rate2–rate1
- **Relative change =** rate2/rate1
- **Proportional (percent) change =** (rate2–rate1)/rate1

Absolute Change in Two Rates

- Example
  - 1989: rate1 = 1,153
  - 1996: rate2 = 307

- **Absolute change** $\Longrightarrow$
  - 307–1,153 = –846 or an absolute decrease in incidence rate of 846 cases per 100,000 person-years
  - (would = 0 if no change)

Relative Change in Two Rates ("Relative Rate")

- Example
  - 1989: rate1 = 1153
  - 1996: rate2 = 307

- **Relative change** $\Longrightarrow$
  - 307 / 1153 = 0.27 or 1–0.27=0.73 or 73% relative decrease in rate
  - (would = 1 if no change)

Proportional Change in Two Rates

- Example
  - 1989: rate1 = 1,153
  - 1996: rate2 = 307

- **Proportional change** $\Longrightarrow$
  - (307–1,153)/1,153 = –.73 or 73% relative decrease in rate
  - (would = 0 if no change)

Summary

- Decision making in public health requires evidence (data)
- Summarizing data as ratios, proportions, and rates
- Commonly used rates
- Concept of person-time
- Assessing change in rates