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Life Tables

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Section A

Clinical Life Tables, Part 1

Clinical Life Table

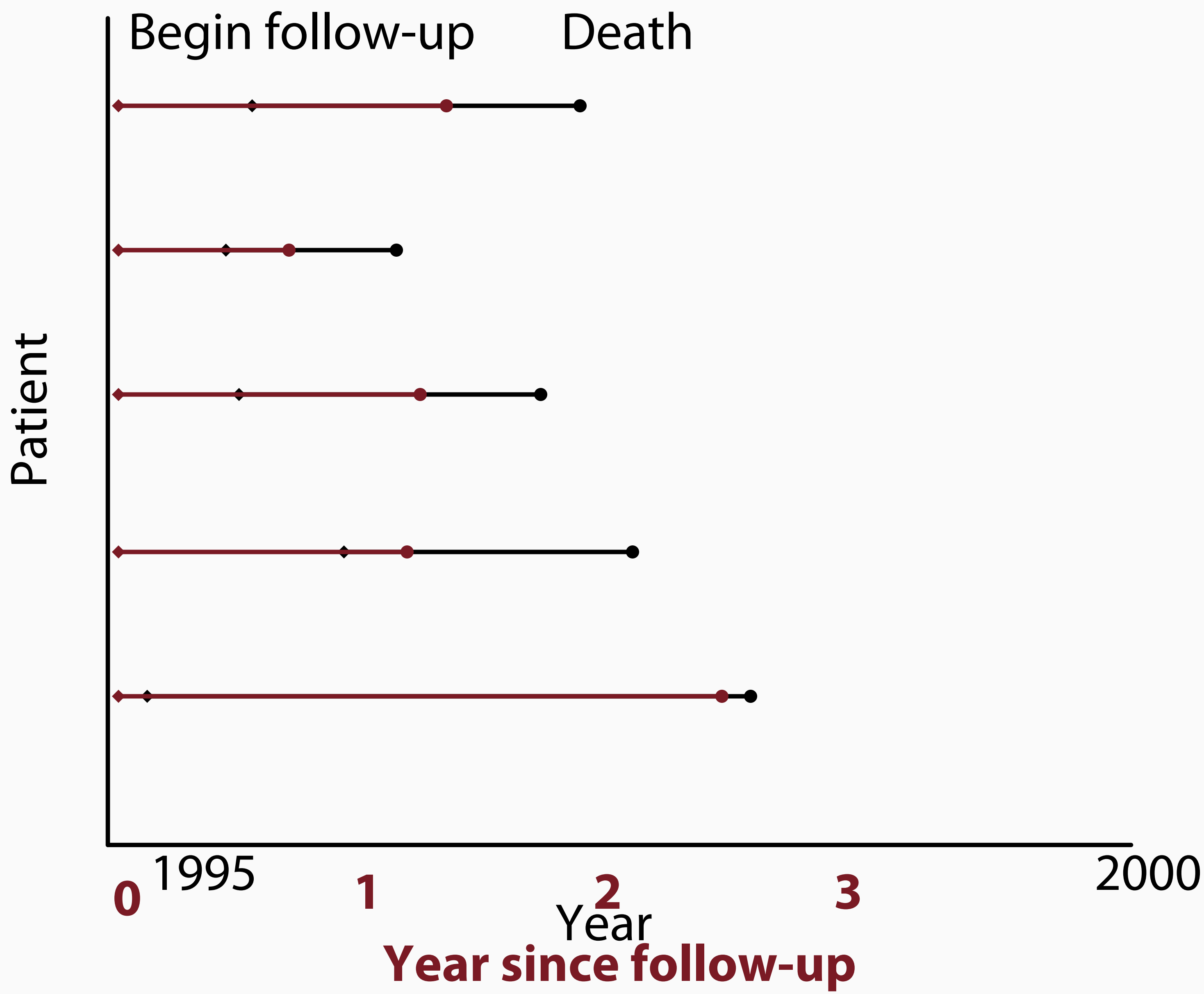
- A fundamental technique of survival analysis that deals with "time to event"
  - A basic example is "time to death"
- It can answer the question of the chance of survival after being diagnosed with the disease or after beginning the treatment
- The event can be any other health event—not just death
  - It can be relapse, receiving organ transplant, pregnancy (in a study of infertility), failure of treatment, recovery, etc.
- It handles variable time of entry and (variable time of) withdrawal of individuals from the population
- It calculates cumulative event-free probabilities and generates a survival curve

Example

- A group of 200 subjects were followed for three years
- Deaths (events) occurred throughout the three years
- What is the chance of surviving at the end of the three years?

Time since beginning of follow-up (Year)	Number at beginning	Deaths			
1	200	20			
2		30			
3		40			

Following a Population



Clinical Life Table Notation

- l_t = number alive at the **beginning** of time *t*
- d_t = number of deaths **during** the time interval

Apply Notation

- Apply notation to the table in the example

Time since beginning of follow-up (Year)	Number at beginning l_t	Deaths d_t			
1	200	20			
2		30			
3		40			

Fill in the "Number at Beginning" Column

- Fill in the missing cells

200–20 = 180

180–30 = 150

Time since beginning of follow-up (Year)	Number at beginning l_t	Deaths d_t			
1	200	20			
2	**180**	30			
3	**150**	40			

Clinical Life Table Notation

- l_t = number alive at the beginning of time t
- d_t = number of deaths during the time interval
- $q_t = d_t / l_t$ = probability of dying during the time interval
- $p_t = 1 - q_t$ = probability of surviving in the time interval

Calculate Probabilities of Dying (q) and Surviving (p)

$20/200 = 0.1$

$30/180 = 0.17$

$1.0 - 0.1 = 0.9$

$1.0 - 0.17 = 0.83$

Interval	l_t	d_t	q_t	p_t	
1	200	20	0.1	0.9	
2	180	30	0.17	0.83	
3	150	40	0.27	0.73	

Clinical Life Table Notation

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- d_t = number of deaths during the time interval
- $q_t = d_t / l_t$ = probability of dying during the time interval
- $p_t = 1 - q_t$ = probability of surviving in the time interval
- P_t = cumulative probability of surviving at the beginning of the time interval
  = cumulative probability of surviving at the end of the previous interval
  - At the beginning of the study (zero time), P(1) = 1.0
  - $P(t+1) = p_t * P_t$

Clinical Life Table Notation

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  = cumulative probability of surviving at the end of the previous interval
  - At the beginning of the study (zero time), P(1) = 1.0
  - $P(t+1) = p_t * P_t$
  - For example: $P_1 = 1.0$

    $P_2 = p_1 * P_1$

    $P_3 = p_2 * P_2$

Calculate the Cumulative Probabilities of Surviving (P)

$P_1 = 1.0$

$P_2 = p_1 * P_1$

$P_3 = p_2 * P_2$

$P_4 = p_3 * P_3$

0.9 x 1.0 = 0.9

0.83 x 0.9 = 0.747

Interval	l_t	d_t	q_t	p_t	P_t
1	200	20	0.1	0.9	1.0
2	180	30	0.17	0.83	0.9
3	150	40	0.27	0.73	0.747

Calculate the Cumulative Probabilities of Surviving (P)

$P_1 = 1.0$

$P_2 = p_1 * P_1$

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0.83 x 0.9 = 0.747

Interval	l_t	d_t	q_t	p_t	P_t
1	200	20	0.1	0.9	1.0
2	180	30	0.17	0.83	0.9
3	150	40	0.27	0.73	0.747
					0.545

0.73 x 0.747 = 0.545

Quick Check

- What is the (cumulative) probability of surviving at the beginning? (time 0) **= 1.0**
- What is the cumulative probability of surviving to the beginning of the second year? **= 0.9**
- What is the cumulative probability of surviving at the end of the first year? **= 0.9**
- What is the cumulative probability of surviving to the beginning of year 3? **= 0.747**
- What is the cumulative probability of surviving to the beginning of year 4 or end of year 3? **= 0.545**

Interval	P_t
1	1.000
2	0.900
3	0.747
	0.545

Section B

Clinical Life Tables: Part 2

Another View of Cumulative Probabilities (P)

- P_t = cumulative probability of surviving at the **beginning** of the time interval

Interval	l_t	d_t	q_t	p_t	p_t
1	200	20	0.1	0.9	1.000
2	180	30	0.17	0.83	0.900
3	150	40	0.27	0.73	0.747
					0.545

1.0 → **0.9**; **0.9** → **0.747**; **0.747** → **0.545**

Censoring

- Observations are considered to be **censored** if:
  - Individuals **withdraw** from the study or are **lost to follow-up**
  - Individuals **are not followed long enough** to experience the event of interest
  - Individuals **experience an event which precludes the event of interest**
- Those who are censored during an interval are assumed to have been followed, on average, for half the interval

Clinical Life Table Notation

- w_t = number withdrew ("censored") during the interval
- $l' = l_t - w_t / 2$ = adjusted number at risk of the event in the interval
- $q_t = d_t / l'_t$

Life Table with Censored Observations

Interval	l_t	d_t	w_t	l'_t	q_t	p_t	P_t
1	200	20	50				
2	130	30	40				
3	60	40	20				

Life Table with Censored Observations

$200 - (50/2) = 200 - 25 = 175$

$20/175 = 0.114$

Interval	l_t	d_t	w_t	l'_t	q_t	p_t	P_t
1	200	20	50	175	0.114	0.886	
2	130	30	40	110	0.273	0.727	
3	60	40	20	50			

Life Table with Censored Observations

20/175 = 0.114

200–(50/2) = 200–25 = 175

Interval	l_t	d_t	w_t	l'_t	q_t	p_t	P_t
1	200	20	50	175	0.114	0.886	1.000
2	130	30	40	110	0.273	0.727	0.886
3	60	40	20	50	0.800	0.200	0.644
							0.129

Clinical Life Table: Assumptions

1. There are no changes in survivorship over calendar time
2. The experience of individuals who are lost to follow-up is the same as the experience of those who are followed

Clinical Life Table: Assumptions

1. There are no changes in survivorship over calendar time
2. The experience of individuals who are lost to follow-up is the same as the experience of those who are followed
3. Withdrawal occurs uniformly within the interval
4. Event occurs uniformly within the interval

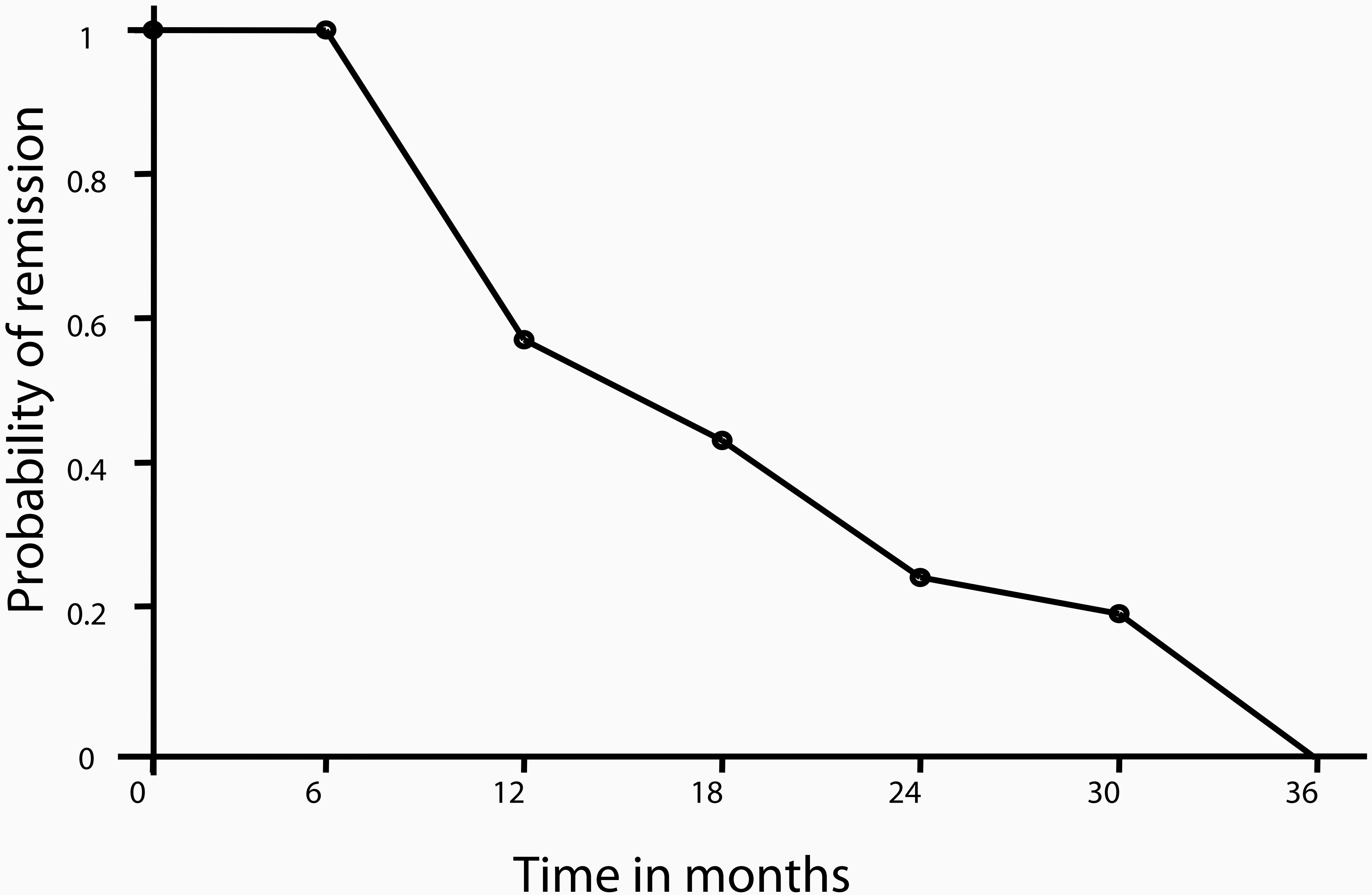
Example 1: Clinical Life Table—No Withdrawal/No Loss

- 21 patients with leukemia were followed after treatment over time
- Time from treatment to relapse was observed for all patients
- The remission times (months) were:
  6, 6, 6, 6, 7, 8, 10, 10, 11, 13, 16, 17, 19, 20, 22, 23, 25, 32, 32, 34,35

Example 1: Clinical Life Table—No Withdrawal/No Loss

Time	l_t	d_t	$q_t = d_t/l_t$	p_t	P_t
0–<6	21	0	0.0000	1.0000	1.0000
6–<12	21	9	0.4286	0.5714	1.0000
12–<18	12	3	0.2500	0.7500	0.5714
18–<24	9	4	0.4444	0.5556	0.4286
24–<30	5	1	0.2000	0.8000	0.2381
30–<36	4	4	1.0000	0.0000	0.1905
					0

Example 1: Plot of Time to Relapse P_t



Example 1: Clinical Life Table—No Withdrawal/No Loss

- P_t = the cumulative probability in remission at time t
  - At the beginning of the time interval
- The cumulative probability still in remission at 24 months is:
  - 0.2381 or 24%
- The probability of relapse between 24 and 30 months is:
  - q_{30} = 0.20 or 20%

Example 2

- 50 patients with skin melanoma were treated in one hospital during the time period October, 1952–June, 1967
- Patients were followed annually
- The study was closed to patient follow-up on December 31, 1969
- 20 deaths occurred
- 30 observations were censored due to withdrawal or lack of follow-up
- What are the **two-year and five-year survival rates?**

Example 2: Clinical Life Table

Interval	l	d	w	l'	q	p	P
0–1	50	9	0	50.0	0.180	0.820	1.000
1–2	41	6	1	40.5	0.148	0.852	0.820
2–3	34	2	4	32.0	0.063	0.937	**0.699**
3–4	28	1	5	25.5	0.039	0.961	0.655
4–5	22	2	3	20.5	0.098	0.902	0.629
5–6	17	0	17	8.5	0	1.000	**0.567**
							0.567

Example 2: Cumulative Probability of Survival

- P is the cumulative **probability** of surviving at the beginning of the time interval or at the end of the previous interval
- Two-year survival **(rate)** is the **probability** of surviving at the end of two years or at the beginning of year 3
- The two-year survival (rate) is 0.699, or 69.9%
- The five-year survival (rate) is 0.567, or 56.7%

Interv al	l	d	w	l'	q	p	P
0–1	50	9	0	50.0	0.180	0.820	1.000
1–2	41	6	1	40.5	0.148	0.852	0.820
2–3	34	2	4	32.0	0.063	0.937	**0.699**
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4–5	22	2	3	20.5	0.098	0.902	0.629
5–6	17	0	17	8.5	0	1.000	**0.567**
							0.567

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Section C

The Kaplan-Meier Method

Kaplan-Meier Method

- **Kaplan-Meier** is also a survival analysis method
  - It is very similar to the clinical life table method
- It uses the exact times that events occurred—rather than the intervals of follow-up
- The probability of the event is equal to the number of events at that time divided by the number at risk at that point in time (including those who had the events)
- If there are withdrawals before the time of event, they are subtracted from the number at risk

Example: Kaplan-Meier Method

- From *Gordis* textbook
- 6 patients
  - 4 died
  - 2 lost to follow-up
- Deaths occurred at 4, 10, 14, and 24 months
- Lost occurred before 10 months and before 24 months

Example: Kaplan-Meier Table and Kaplan-Meier Plot

- The table is similar to the clinical life table
  - Instead of intervals, it uses the exact time of events
- In this example, the events occurred at 4, 10, 14, and 24 months (so there will be 4 rows in the table)
- All other calculations (q, p, and P) are the same
- The calculated cumulative probability of surviving is for that time point
- Up to that time point, the cumulative probability of surviving takes on the value of the previous time point, thus leading to a step function (see K-M plot)
- When there is no event, the survival curve in a K-M plot will be drawn out horizontally over time and only drop (vertically) down at the time of events (e.g., deaths) to the calculated cumulative probability of surviving

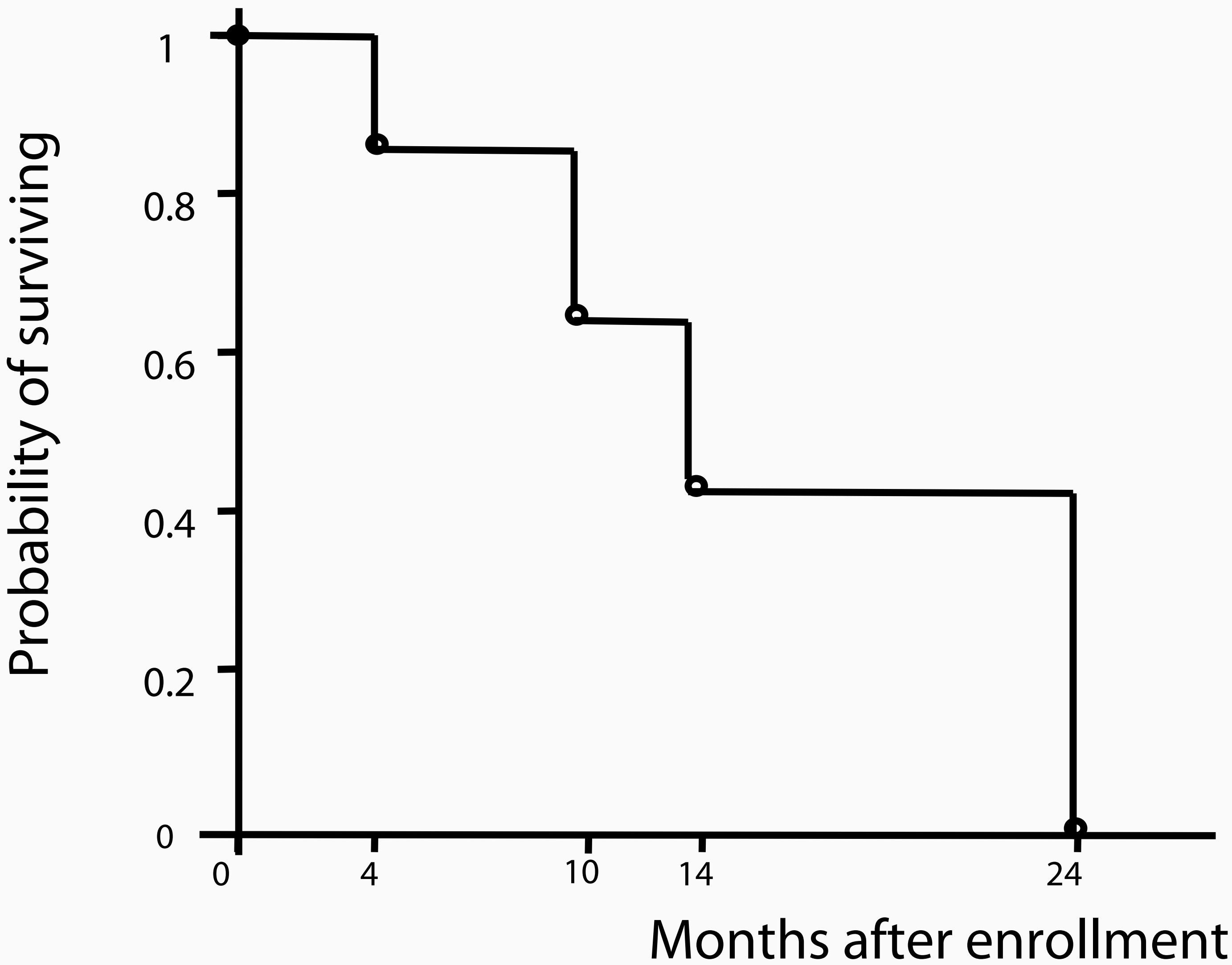
Kaplan-Meier Survival Table

One died at 4 months, and one was lost to follow-up before 10 months; therefore, 4 were known to be alive at 10 months

Time to deaths	Number alive	d_t	q_t	p_t	P_t
4	6	1	0.167	0.833	0.833
10	4	1	0.250	0.750	0.625
14	3	1	0.333	0.667	0.417
24	1	1	1.000	0.000	0.000

Another "lost" occurred here before 24 months

Kaplan-Meier Plot of Survival Study



Use of Kaplan-Meier Method

- K-M method takes advantage of all information available in the calculation and is useful for small sample size studies
- Clinical studies use K-M plots to display prognosis over time
- K-M estimates can be used for comparison purpose in clinical trials when groups are similar and adjustment is not needed

Review

- What is the difference between **l** and **l'**?
- What is the difference between **p** and **P**?
- What are the assumptions in a clinical life table?