This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike License. Your use of this material constitutes acceptance of that license and the conditions of use of materials on this site.

JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Copyright 2009, The Johns Hopkins University and John McGready. All rights reserved. Use of these materials permitted only in accordance with license rights granted. Materials provided "AS IS"; no representations or warranties provided. User assumes all responsibility for use, and all liability related thereto, and must independently review all materials for accuracy and efficacy. May contain materials owned by others. User is responsible for obtaining permissions for use from third parties as needed.

JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Section F

The Theoretical Sampling Distribution of the Sample Proportion and Its Estimate Based on a Single Sample

Sampling Distribution of the Sample Mean

- In the previous section we reviewed the results of simulations that resulted in estimates of what was formally called the sampling distribution of a sample proportion

- The sampling distribution of a sample proportion is a theoretical probability distribution
  - It describes the distribution of all sample proportions from all possible random samples of the same size taken from a population

Sampling Distribution of the Sample Mean

- In real research it is impossible to estimate the sampling distribution of a sample mean by actually taking multiple random samples from the same population (no research would ever happen if a study needed to be repeated multiple times) to understand this sampling behavior

- Simulations are useful to illustrate a concept, but not to highlight a practical approach!

- Luckily, there is some mathematical machinery that generalizes some of the patterns we saw in the simulation results

The Central Limit Theorem (CLT)

- The Central Limit Theorem (CLT) is a powerful mathematical tool that gives several useful results
  - The sampling distribution of sample proportions based on all samples of same size *n* is approximately normal
  - The mean of all sample proportions in the sampling distribution is the true mean of the population from which the samples were taken, *p*
  - The standard deviation in the sample proportions of size *n* is equal to $\sqrt{\frac{p \times (1-p)}{n}}$
  - This is often called the standard error of the sample proportion and sometimes written as $SE(\hat{p})$

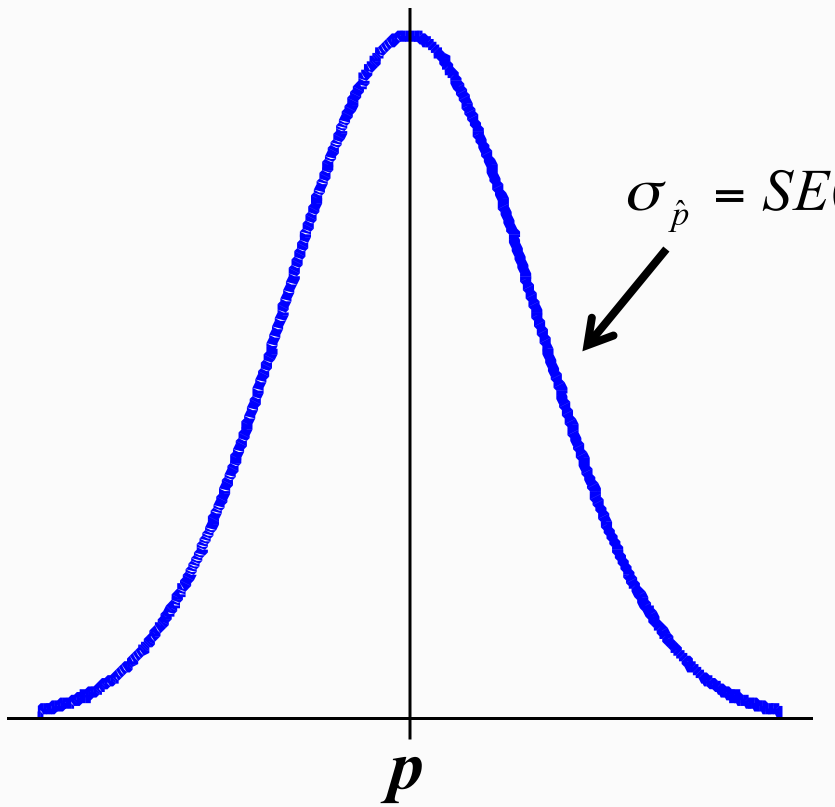
Example: Blood Pressure of Males

- Population distribution of individual insurance status
  - True proportion $p = 0.8$

Sample Sizes	Means of 500 Sample Proportions	Means of 5000 Sample Proportions	SD of 500 Sample Proportion	SD of 5000 Sample Proportions	SD of Sample Proportions (SE) by CLT
n = 20	**0.805**	**0.799**	**0.094**	**0.090**	**0.089**
n = 100	**0.801**	**0.799**	**0.041**	**0.040**	**0.040**
n = 1,000	**0.799**	**0.80**	**0.012**	**0.012**	**0.012**

Recap: CLT

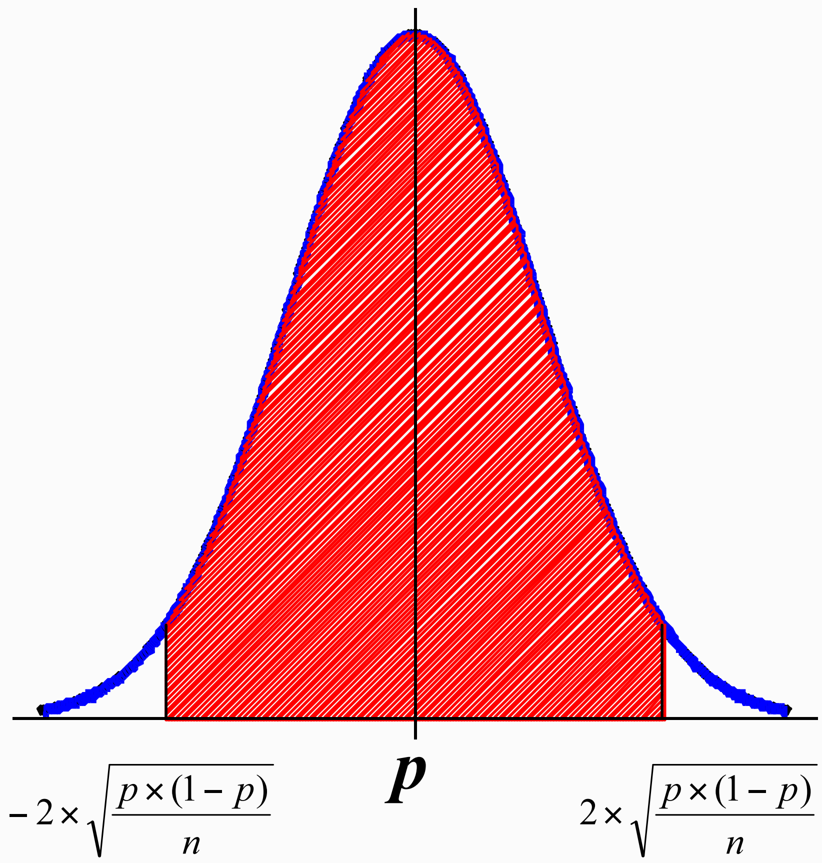
- So the CLT tells us the following:
  - When taking a random sample of binary measures of size *n* from a population with true proportion *p* the theoretical sampling distribution of sample proportions from all possible random samples of size *n* is:



$$\sigma_{\hat{p}} = SE(\hat{p}) = \sqrt{\frac{p \times (1-p)}{n}}$$

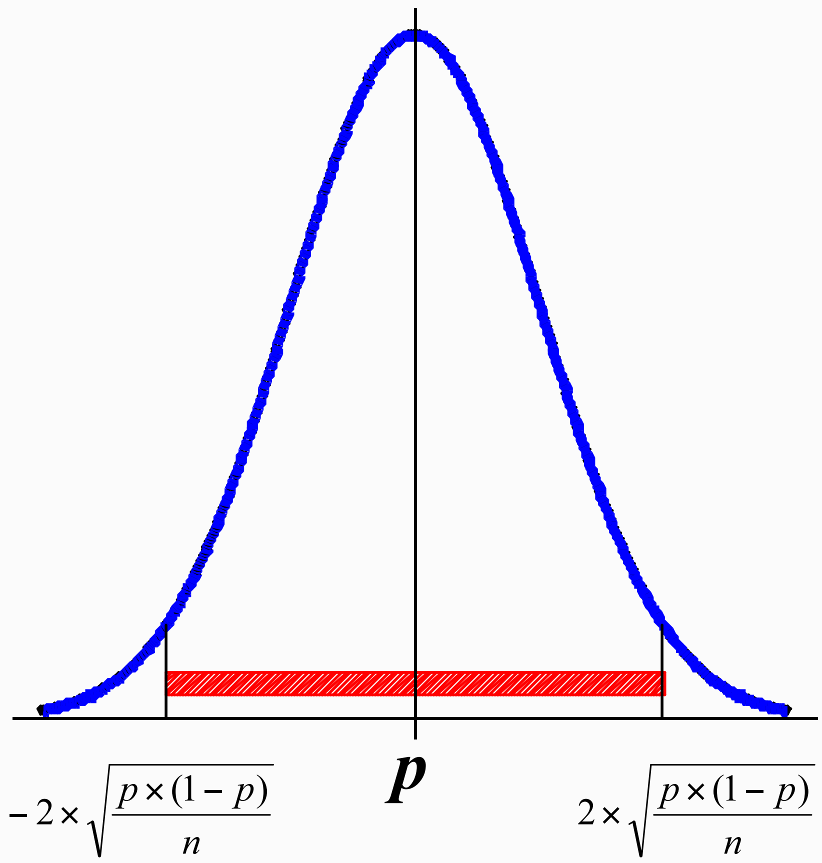
CLT: So What?

- So what good is this info?
  - Well using the properties of the normal curve, this shows that for most random samples we can take (95%), the sample proportion $\hat{p}$ will fall within 2 SEs of the true proportion *p*:



CLT: So What?

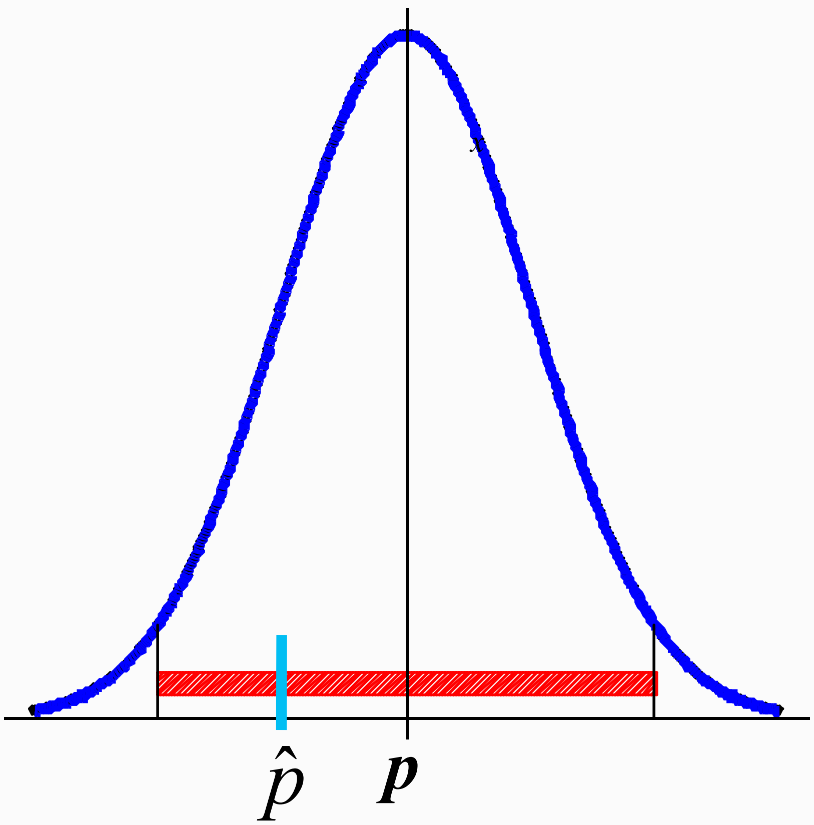
- So AGAIN what good is this info?
  - We are going to take a single sample of size *n* and get one $\hat{p}$
  - So we won't know *p* and if we did know *p* why would we care about the distribution of estimates of *p* from imperfect subsets of the population?



CLT: So What?

- We are going to take a single sample of size *n* and get one $\hat{p}$

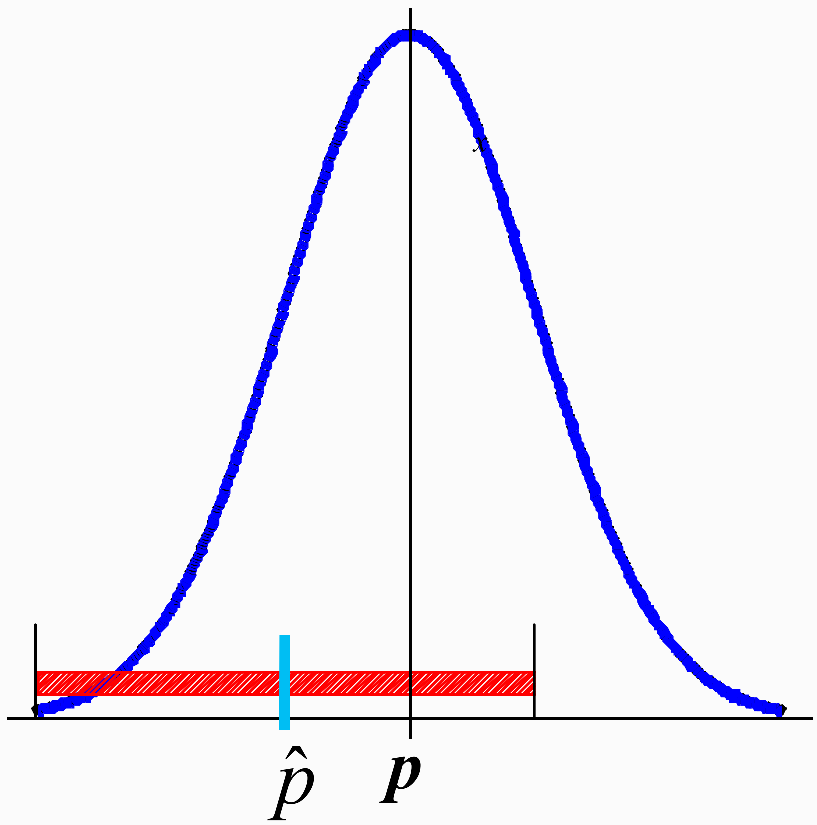
- But for most (95%) of the random samples we can get, our $\hat{p}$ will fall within +/- 2 SEs of *p*



CLT: So What?

- We are going to take a single sample of size *n* and get one $\hat{p}$

- So if we start at $\hat{p}$ and go 2 SEs in either direction, the interval created will contain *p* most (95 out of 100) of the time



Estimating a Confidence Interval

- Such an interval is called a 95% confidence interval for the population proportion *p*
- Interval given by $\hat{p} \pm 2SE(\hat{p}) \rightarrow \bar{x} \pm 2 \times \sqrt{\frac{p \times (1 - p)}{n}}$
- Problem: we don't know *p*
  - Can estimate with $\hat{p}$, will detail this in next section
- What is interpretation of a confidence interval?

Interpretation of a 95%Confidence Interval (CI)

- Laypersons' range of "plausible" values for true proportion
  - Researcher never can observe true mean *p*
  - $\hat{p}$ is the best estimate based on a single sample
  - The 95% CI starts with this best estimate and additionally recognizes uncertainty in this quantity

- Technical
  - Were 100 random samples of size *n* taken from the same population, and 95% confidence intervals computed using each of these 100 samples, 95 of the 100 intervals would contain the values of true proportion *p* within the endpoints

Notes on Confidence Intervals

- Random sampling error
  - Confidence interval only accounts for random sampling error, not other systematic sources of error or bias

Notes on Confidence Intervals

- Are all CIs 95%?
  - No
  - It is the most commonly used
  - A 99% CI is wider
  - A 90% CI is narrower

- To change level of confidence adjust number of SE added and subtracted from $\hat{p}$
  - For a 99% CI, you need ± 2.6 SE
  - For a 95% CI, you need ± 2 SE
  - For a 90% CI, you need ± 1.65 SE

Summary

- What did we see with this set of examples

- A couple of trends:
  - Distribution of sample proportions tended to be approximately normal—even when original—and individual level data was not (binary outcome)
  - Variability in sample mean values decreased as the size of the sample each proportion was based upon increased

Clarification

- As with means for continuous data, variation in proportions values tied to the size of each sample selected in our exercise: NOT the number of samples