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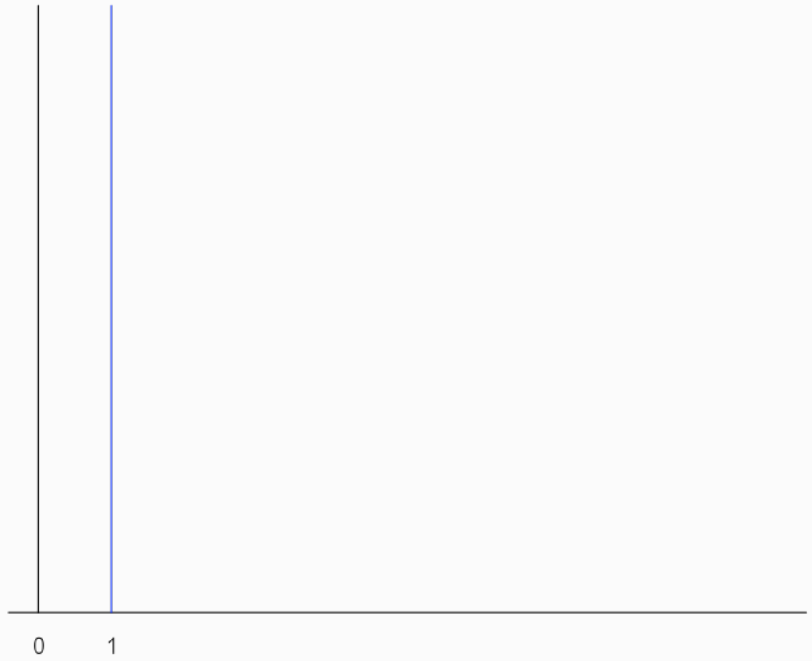
SCHOOL of PUBLIC HEALTH

Section G

FYI: Sampling Behavior of Relative Risks/Odds Ratios

Sampling Behavior of Ratios Is Not Normal

- The sampling behavior of ratios (like the RR, OR) can be quite skewed
  - The range of possible values for “positive” and “negative” associations are very different



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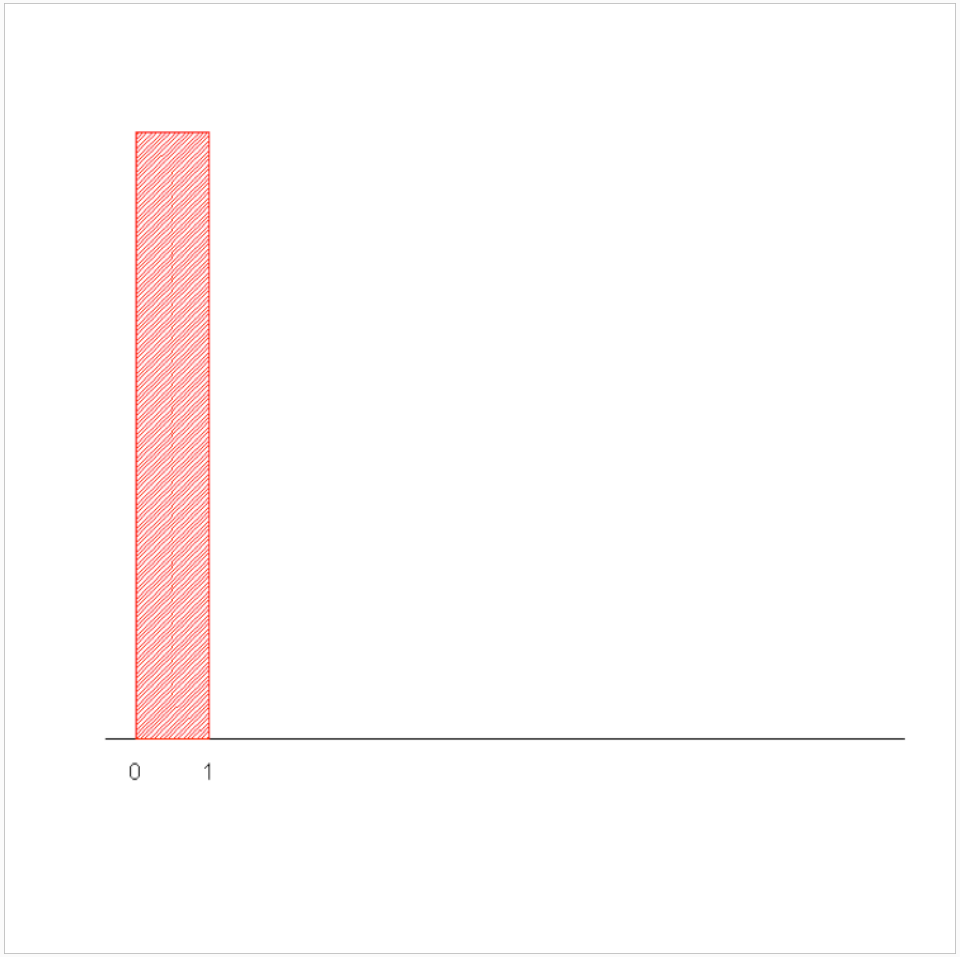

Sampling Behavior of Ratios Is Not Normal

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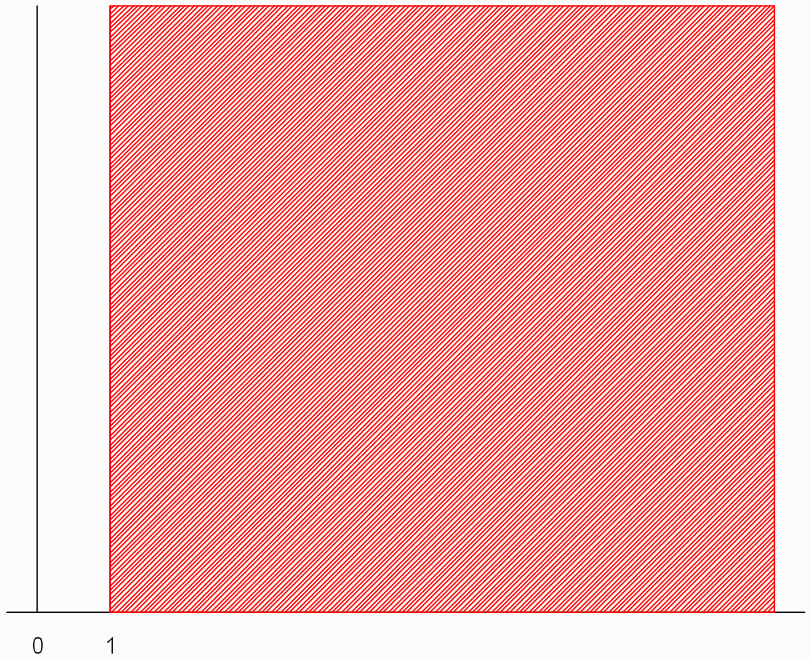
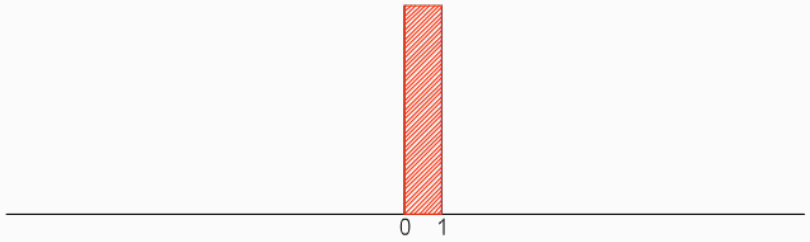
  - The range of possible values for “positive” associations



Sampling Behavior of Ratios Is Not Normal

- The ranges are equal on the ln(Ratio) scale



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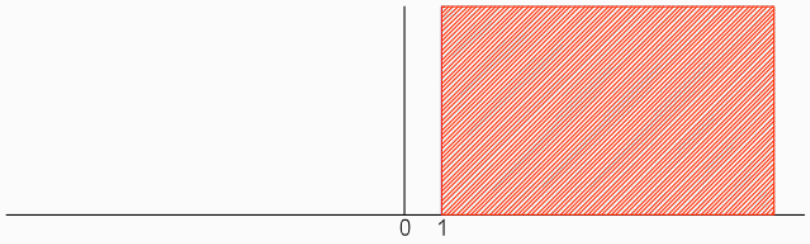

Sampling Behavior of Ratios Is Not Normal

- Recall standard 2x2 table setup

		Exposure: Yes	Exposure: No
Outcome	Yes	a	b
	No	c	d

Estimating CI for RR by Hand

- ln ratios and standard errors

$$ln(\hat{RR}) = ln(\frac{\hat{p}_1}{\hat{p}_2})$$

- Standard error, using counts from 2x2 table

$$\hat{SE}(ln(\hat{RR})) = \sqrt{\frac{c}{a \times n_1} + \frac{d}{b \times n_2}}$$

- 95% CI for $ln(RR)$

$$ln(\hat{RR}) \pm 2 \times \hat{SE}(ln(\hat{RR}))$$

- To get 95% CI for RR, exponentiate endpoints of above

HIV/AZT Example

- HIV/mother-infant transmission example

		Drug Group		
		AZT	Placebo	
HIV Transmission	Yes	13	40	53
	No	167	143	310
		180	183	363

Sampling Behavior of Ratios Is Not Normal

- ln ratios and standard errors

$$ln(\hat{RR}) = ln(\frac{\hat{p}_{AZT}}{\hat{p}_{Placebo}}) = ln(\frac{.07}{.22}) = ln(0.33) = -1.11$$

- Standard error, using counts from 2x2 table

$$\hat{SE}(ln(\hat{RR})) = \sqrt{\frac{167}{13 \times 180} + \frac{143}{40 \times 183}} \approx .30$$

- 95% CI for $ln(RR)$

$$-1.11 \pm 2 \times .30 \rightarrow (-1.71, -0.51)$$

- To get 95% CI for RR, exponentiate endpoints of above

$$(e^{-1.71}, e^{-0.51}) \approx (0.18, 0.60)$$

HIV/AZT Example

- 95% CI from Stata

```
. csi 13 40 167 143, or

                 |   Exposed   Unexposed  |     Total
-----------------+------------------------+----------
           Cases |        13          40  |        53
        Noncases |       167         143  |       310
-----------------+------------------------+----------
           Total |       180         183  |       363
                 |                        |
            Risk |  .0722222    .2185792  |  .1460055
                 |                        |
                 |      Point estimate    |  [95% Conf. Interval]
                 |------------------------+----------------------
Risk difference  |         -.146357       | -.2171766   -.0755374
     Risk ratio  |         .3304167       |  .1829884    .5966235
Prev. frac. ex.  |         .6695833       |  .4033765    .8170116
Prev. frac. pop  |         .3320248       |
     Odds ratio  |         .2782934       |  .1445784    .5363045  (Cornfield)
                 +-----------------------------------------------
                           chi2(1) =    15.59  Pr>chi2 = 0.0001
```

Sampling Behavior of Ratios Is Not Normal

- ln ratios and standard errors

$$ln(\hat{OR}) = ln(\frac{\hat{p}_1/(1-\hat{p}_1)}{\hat{p}_2/(1-\hat{p}_2)})$$

- Standard error, using counts from 2x2 table

$$\hat{SE}(ln(\hat{OR})) = \sqrt{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

- 95% CI for $ln(\hat{OR})$

$$ln(\hat{OR}) \pm 2 \times \hat{SE}(ln(\hat{OR}))$$

- To get 95% CI for OR, exponentiate endpoints of above

Sampling Behavior of Ratios is Not Normal

- HIV/AZT transmission example

$$ln(\hat{OR}) = ln(\frac{.07/.93}{.22/.78}) \approx \ln(.28) = -1.27$$

- Standard error, using counts from 2x2 table

$$\hat{SE}(ln(\hat{OR})) = \sqrt{\frac{1}{13} + \frac{1}{40} + \frac{1}{167} + \frac{1}{143}} \approx .34$$

- 95% CI for $ln(\hat{OR})$

$$-1.27 \pm 2 \times .34 \rightarrow (-1.96, -0.60)$$

- To get 95% CI for OR, exponentiate endpoints of above

$$(e^{-1.96}, e^{-0.60}) \approx (0.14, 0.55)$$

HIV/AZT Example

- 95% CI from Stata

```
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                 |   Exposed   Unexposed  |     Total
-----------------+------------------------+----------
           Cases |        13          40  |        53
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           Total |       180         183  |       363
                 |                        |
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                 |                        |
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```