JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Lecture 16

Brian Caffo

Department of Biostatistics
Johns Hopkins Bloomberg School of Public Health
Johns Hopkins University

December 19, 2007

Table of contents

Outline

1. Power
2. Power for a one sided normal test
3. Power for t-test

Power

- Power is the probability of rejecting the null hypothesis when it is false
- Ergo, power (as it's name would suggest) is a good thing; you want more power
- A type II error (a bad thing, as its name would suggest) is failing to reject the null hypothesis when it's false; the probability of a type II error is usally called β
- Note Power $= 1 - \beta$

Notes

- Consider our previous example involving RDI
- $H_0 : \mu = 30$ versus $H_a : \mu > 30$
- Then power is

$$P\left(\frac{\bar X - 30}{s/\sqrt{n}} > t_{1-\alpha,n-1} \mid \mu = \mu_a\right)$$

- Note that this is a function that depends on the specific value of μ_a!
- Notice as μ_a approaches 30 the power approaches α

Calculating power

Assume that n is large and that we know σ

$$
\begin{aligned}
1-\beta &= P\left(\frac{\bar X - 30}{\sigma/\sqrt{n}} > z_{1-\alpha} ~|~ \mu = \mu_a\right)\\
&= P\left(\frac{\bar X - \mu_a + \mu_a - 30}{\sigma/\sqrt{n}} > z_{1-\alpha} ~|~ \mu = \mu_a\right)\\
&= P\left(\frac{\bar X - \mu_a}{\sigma/\sqrt{n}} > z_{1-\alpha} - \frac{\mu_a - 30}{\sigma/\sqrt{n}} ~|~ \mu = \mu_a\right)\\
&= P\left(Z > z_{1-\alpha} - \frac{\mu_a - 30}{\sigma/\sqrt{n}} ~|~ \mu = \mu_a\right)
\end{aligned}
$$

Example continued

- Suppose that we wanted to detect a increase in mean RDI of at least 2 events / hour. Assume normality and that the sample in question will have a standard deviation of 4; what would be the power if we took a sample size of 16?
- $Z_\alpha = 1.645$ and $\frac{\mu_a - 30}{\sigma/\sqrt{n}} = 2/(4/\sqrt{16}) = 2$
- $P(Z > 1.645 - 2) = P(Z > -0.355) = 64\%$

Example continued

- What n would be required to get a power of 80%
- I.e. we want

$$0.80 = P\left(Z > z_{1-\alpha} - \frac{\mu_a - 30}{\sigma/\sqrt{n}} \mid \mu = \mu_a\right)$$

- Set $z_{1-\alpha} - \frac{\mu_a - 30}{\sigma/\sqrt{n}} = z_{0.20}$ and solve for n

Notes

- The calculation for $H_a : \mu > \mu_0$ is similar
- For $H_a : \mu \neq \mu_0$ calculate the one sided power using $\alpha/2$ (this is only approximately right, it excludes the probability of getting a large TS in the opposite direction of the truth)
- Power goes up as α gets larger
- Power of a one sided test is greater than the power of the associated two sided test
- Power goes up as μ_1 gets further away from μ_0
- Power goes up as n goes up

Power for the T test

- Consider calculating power for a Gossett's T test for our example
- The power is

$$P\left(\frac{\bar X - 30}{S/\sqrt{n}} > t_{1-\alpha,n-1} \mid \mu = \mu_a\right)$$

- Notice that this is equal to

$$= P\left(\sqrt{n}(\bar X - 30) + > t_{1-\alpha,n-1} S \mid \mu = \mu_a\right)$$

$$= P\left(\frac{\sqrt{n}(\bar X - 30)}{\sigma} + > t_{1-\alpha,n-1} \frac{S}{\sigma} \mid \mu = \mu_a\right)$$

Continued

- Continued

$$P\left(\frac{\sqrt{n}(\bar{X}-\mu_a)}{\sigma}+\frac{\sqrt{n}(\mu_a-30)}{\sigma}>\frac{t_{1-\alpha,n-1}}{\sqrt{n-1}}\times\sqrt{\frac{(n-1)S^2}{\sigma^2}}\right)$$

(where we ommitted the conditional on μ_a part for space)

- This is now equal to

$$P\left(Z+\frac{\sqrt{n}(\mu_a-30)}{\sigma}>\frac{t_{1-\alpha,n-1}}{\sqrt{n-1}}\sqrt{\chi^2_{n-1}}\right)$$

where Z and χ^2_{n-1} are independent standard normal and chi-squared random variables

- While computing this probability is outside the scope of the class, it would be easy to approximate with Monte Carlo

Example

Let's recalculate power for the previous example using the T distribution instead of the normal; here's the easy way to do it. Let $\sigma = 4$ and $\mu_a - \mu_0 = 2$

```
##the easy way
power.t.test(n = 16, delta = 2 / 4,
             type = "one.sample",
             alt = "one.sided")
##result is 60%
```

Example

Using Monte Carlo

```
nosim <- 100000
n <- 16
sigma <- 4
mu0 <- 30
mua <- 32
z <- rnorm(nosim)
xsq <- rchisq(nosim, df = 15)
t <- qt(.95, 15)
mean(z + sqrt(n) * (mua - mu0) / sigma >
     t / sqrt(n - 1) * sqrt(xsq))
##result is 60%
```

Comments

- Notice that in both cases, power required a true mean and a true standard deviation
- However in this (and most linear models) the power depends only on the mean (or change in means) divided by the standard deviation