Outline

1. Describe multivariate Bernoulli trials
2. Motivate the multinomial distribution

Multinomial density

- The multinomial distribution is a generalization of the binomial distribution where each trial can take several levels, rather than just two
- To illustrate the multinomial distribtion, consider drawing with replacement from an an urn with with blue, red and yellow balls

$(1,0,0)$ for getting a blue ball
$(0,1,0)$ for getting a red ball
$(0,0,1)$ for getting a yellow ball

Continued

- Assume that the proportion of blue, red and yellow balls in the urn are π_1,π_2 and π_3
- $\pi_1 + \pi_2 + \pi_3 = 1$

$$P\{X_i = (1,0,0)\} = \pi_1$$
$$P\{X_i = (0,1,0)\} = \pi_2$$
$$P\{X_i = (0,0,1)\} = \pi_3.$$

- Notice that one of the numbers is redundant. For example, if we know that the outcome is neither a blur or a red ball, then it must have been a yellow ball
- We might call this generalization **the multivariate Bernoulli distribution**

Continued

- Just as a binomial random variable is the sum of iid Bernoulli trials, the multinomial distribution is the sum of iid multivariate Bernoulli trials
- Therefore, in our urn example, if you sum up n multivariate Bernoullis from this experiment, you get a vector that looks like

$$(n_1, n_2, n_3)$$

  where n_1, n_2 and n_3 are the number of blue, red and yellow balls (and $n_1 + n_2 + n_3 = n$)

Some properties

- Mass function

$$P\{(N_1, \ldots, N_k) = (n_1, \ldots, n_k)\} = \frac{n!}{\prod_{i=1}^{k} n_i!} \prod_{i=1}^{k} \pi_i^{n_i}$$

- $E[N_i] = n\pi_i$
- $\mathrm{Var}(N_i) = n\pi_i(1 - \pi_i)$
- $\mathrm{Cov}(N_i, N_j) = -n\pi_i\pi_j$
- The maximum likelihood estimate of π_i is N_i/n; that is the sample proportion remains the best estimate of the population proportion.