Lecture 3d: Practice Problem Solutions

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1. A healthcare information company is interested in estimating the average charge for a standard patient visit to a chiropractor in Maryland, after applying the discount negotiated with a large HMO plan. Data is collected from 16 randomly selected chiropractic practices in Maryland, and the following are some summary statistics:
   - Mean charge: 25.50 USD
   - SD of charges: 2.10 USD
1. Assuming the charge data is normally distributed for all chiropractic practices in Maryland, estimate a range of amounts that most (95%) of the chiropractic practices in Maryland charge for a standard patient visit
   - If the sample data comes from a distribution of normally distributed values, then we can estimate this interval from the sample results by using $\bar{x} \pm 2 \times s$.
   - In this example, applying this logic gives an interval of
     $$25.50 \pm 2 \times 2.10 \rightarrow 25.50 \pm 4.20 \rightarrow ($21.30, $29.70).$$
2. Without assuming normality, estimate a range of amounts that most (95%) of the chiropractic practices in Maryland charge for a standard patient visit
   - This is a trick question: given only the sample mean and standard deviation, it is not possible to answer this question
3. Assuming the charge data is normally distributed for all chiropractic practices in Maryland, estimate a 95% confidence interval for the mean amount charged by Maryland chiropractors.

- We can go ahead and estimate a 95% CI for the true average charge among all Maryland chiropractors using the formula,

\[
\bar{x} \pm t_{0.95,16-1} \times \frac{SE(\bar{x})}{\sqrt{n}}
\]

- (i.e., \(\bar{x} \pm 2.13 \times \frac{s}{\sqrt{n}}\))

- For these sample results, this gives:

\[
25.50 \pm 2.13 \times \frac{2.10}{\sqrt{16}} \rightarrow 25.50 \pm 1.12 \rightarrow
\]

\($24.38, \$26.62)\)
Estimating a 95% Confidence Interval

4. Without assuming normality, estimate a 95% confidence interval for the mean amount charged by Maryland chiropractors
   - Actually, the normality assumption about the original individual data is NOT needed to use our results to estimate 95% CI for the true underlying population mean of the population from which the sample was taken
   - Hence the answer to this question is EXACTLY the same as the answer to the previous question (3)
   - This highlights one of the powers of the CLT related results: that we can estimate a valid confidence interval for a population mean using only a single random sample of data, regardless of what the population distribution (and random sample distribution) of the individual level data is
Estimating a 95% Confidence Interval

5. What is the difference in the interpretation of the intervals created in questions 1 and 2 and questions 3 and 4?
   - Same answer as problem 5 from lecture 3C practice problems