JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Lecture 4c: Practice Problem Solutions

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Practice: Paired t-test

1. Eight counties were selected from State A
   - Each of these counties was matched with a county from State B, based on factors:
     - Mean income
     - Percentage of residents living below the poverty level
     - Violent crime rate
     - Infant Mortality Rate (IMR) in 2006
     - Information on the infant mortality rate in 2007 was collected on each set of eight counties (IMR is measured in deaths per 10,000 live births)
     - A pre- and post-neonatal care program was implemented in State B at the beginning of 2007

Practice: Paired t-test

1. This data is being used to compare the IMR rates in States A and B in 1997
   - This comparison will be used as part of the evaluation of the neonatal care program in State B, regarding its effectiveness on reducing infant mortality

Practice: Paired t-test

1. What is the appropriate method for testing whether the mean IMR is the same for both states in 2007?
   a) Estimate a 95% confidence for the true mean difference in IMR between the two state groupings

   – We can use the formula $\bar{x}_{diff} \pm t_{0.95,7} \times \hat{SE}(\bar{x}_{diff})$

   – i.e., $\bar{x}_{diff} \pm 2.37 \times \frac{s_{diff}}{\sqrt{n}}$.

   – With this data the resulting 95% CI is given by

$$6.1 \pm 2.37 \times \frac{14.5}{\sqrt{8}} \approx 6.1 \pm 12.2 \rightarrow (-6.1, 18.3)$$

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b) State your null and alternative hypotheses for the corresponding hypothesis test

- H_o: $\mu_{diff} = 0$
- H_o: $\mu_{diff} \neq 0$

- Where μ_{diff} represents the difference in average IMR rates between all possible pairings of counties from states A and B

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c) Report a p-value for the hypothesis test
   - "By hand": the distance measure,
   - $$t = \frac{\bar{x}_{diff} - \mu_o}{S\hat{E}(\bar{x}_{diff})} = \frac{6.1 - 0}{14.5/\sqrt{8}} = 1.19.$$
   - Our sample result is 1.19 standard errors above 0. How likely/ unlikely? (you'll need to consult a t-table with seven degrees of freedom)

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c) Report a p-value for the hypothesis test
   - Better approach, use Stata!
   - Gives p-value of .27

```
. ttesti 8 6.1 14.5 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |       8         6.1    5.126524        14.5   -6.022303     18.2223
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   1.1899
Ho: mean = 0                                     degrees of freedom =        7

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.8636         Pr(|T| > |t|) = 0.2729          Pr(T > t) = 0.1364
```

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d) Do the results from the 95% confidence interval and the p-value agree in terms of the null hypothesis (using $\alpha = .05$ for the hypothesis test)
   - Yes. The 95% for the true mean difference includes 0, and the p-value for testing the null that this true difference is 0 is .27, which would not be rejected in favor of the alternative hypothesis with an $\alpha=.05$ rejection level

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e) What would your results be for A - D if you had 32 county pairs, and the mean change and standard deviation of the differences were the same ?

- You could do this by hand but for the sake of brevity here is Stata output:

```
. ttesti 32 6.1 14.5 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      32         6.1    2.563262        14.5    .8721925    11.32781
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   2.3798
Ho: mean = 0                                     degrees of freedom =       31

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.9882         Pr(|T| > |t|) = 0.0237          Pr(T > t) = 0.0118
```

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2. What is the role of the α-level in hypothesis testing?
   - The α-level is the cutoff for calling a p-value "statistically significant" or not. It is the threshold set by the researcher to determine whether a result is consistent with the null ("likely" when the null is true) or inconsistent with the null ("unlikely" when the null is true). This value does not depend on any data and can be set at any time in the research process. The "industry standard" is .05 (5%).

Practice: Paired t-test

3. What is the role of the p-value in hypothesis testing?
   - The p-value provides a measure of how likely (/unlikely) a sample result and other results less likely) under an assumed truth, the null hypothesis. This p-value can be compared to a preset rejection cutoff (α-level) in order to make a decision to either reject or fail to reject this assumed null hypothesis as a possibility for the truth.

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4. Seventy individuals were enrolled in a dietary counseling program intended to promote healthier eating. Each subject had his/her sodium levels measured on the day of enrollment and after two weeks of counseling. The results of these measurements were as follows:

$$\bar{x}_{pre} = 17.7\ mEq/8hr$$

$$\bar{x}_{post} = 16.5\ mEq/8hr$$

$$s_{difference} = 12.2\ mEq/8hr$$

   – Was the change in average sodium excretion statistically significant at the .05 level? Justify your answer numerically.

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5. First, you need to compute sample mean difference

$$\bar{x}_{post} - \bar{x}_{pre} = 16.5 - 17.7 = -1.2\ mEq/8hr$$

– Now we have everything we need to do the computations. I'll show the results in Stata, see if you can verify by hand.

Practice: Paired t-test

5. Stata results

```
. ttesti 70 -1.2 12.2 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      70        -1.2    1.458179        12.2   -4.108987    1.708987
------------------------------------------------------------------------------
    mean = mean(x)                                                t =  -0.8229
Ho: mean = 0                                     degrees of freedom =       69

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.2067         Pr(|T| > |t|) = 0.4134          Pr(T > t) = 0.7933
```

Practice: Paired t-test

5. Suppose you had computed the mean difference in the opposite direction (pre – post).

$$\bar{x}_{pre} - \bar{x}_{post} = 17.7 - 16.5 = 1.2\ mEq/8hr$$

   - How would this change your results?

Practice: Paired t-test

5. Stata results, opposite direction of comparison

```
. ttesti 70 1.2 12.2 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      70         1.2    1.458179        12.2   -1.708987    4.108987
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   0.8229
Ho: mean = 0                                     degrees of freedom =       69

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.7933         Pr(|T| > |t|) = 0.4134          Pr(T > t) = 0.2067
```