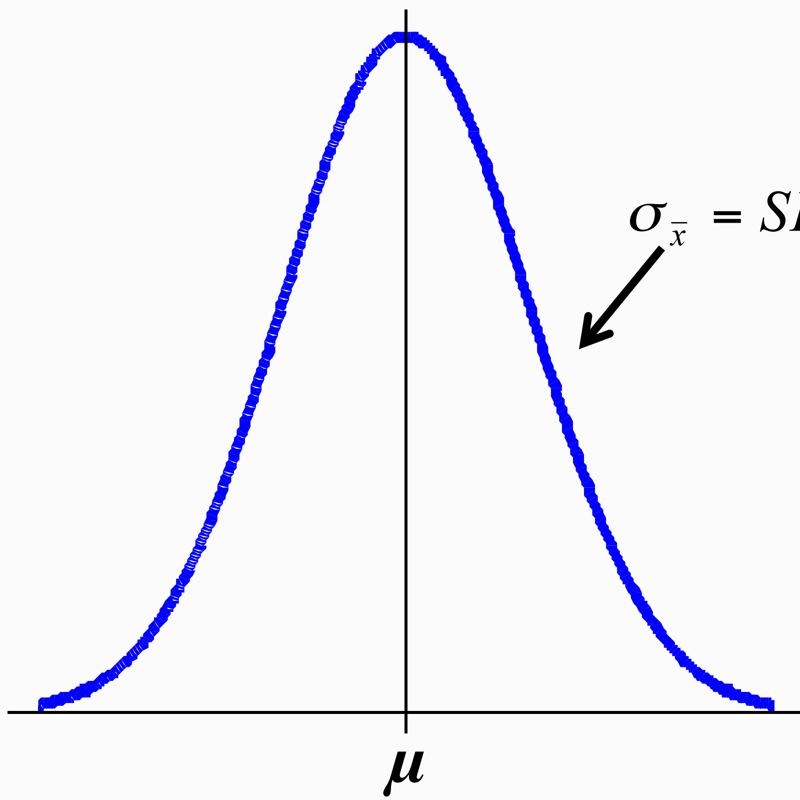
JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Section D

True Confessions Biostat Style: What We Mean by Approximately Normal and What Happens to the Sampling Distribution of the Sample Mean with Small *n*

Recap: CLT

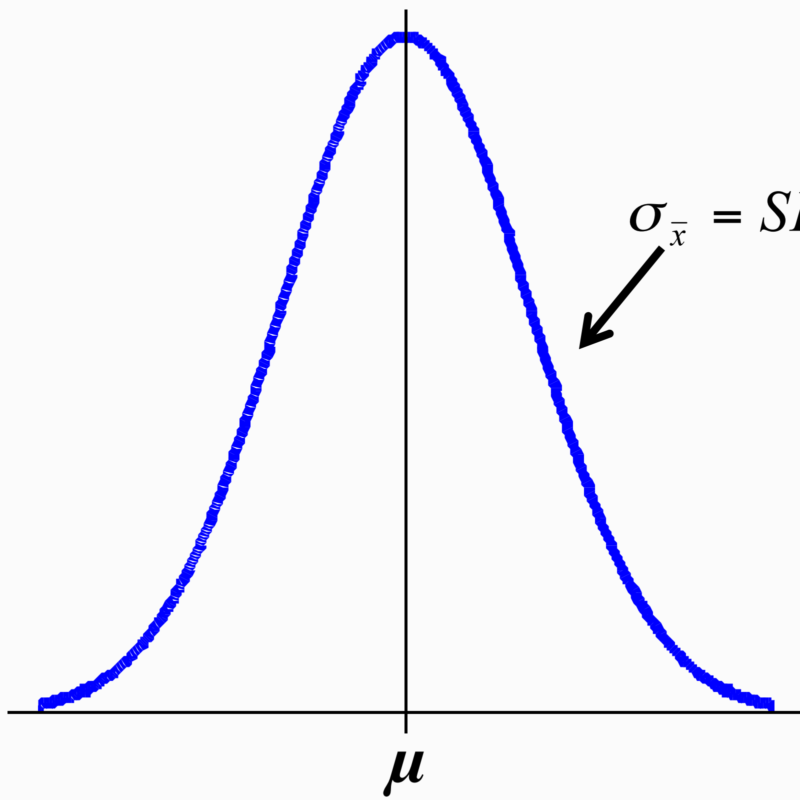
- So the CLT tells us the following: when taking a random sample of continuous measures of size *n* from a population with true mean μ and true sd σ the theoretical sampling distribution of sample means from all possible random samples of size *n* is:



$$\sigma_{\bar{x}} = SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

Recap: CLT

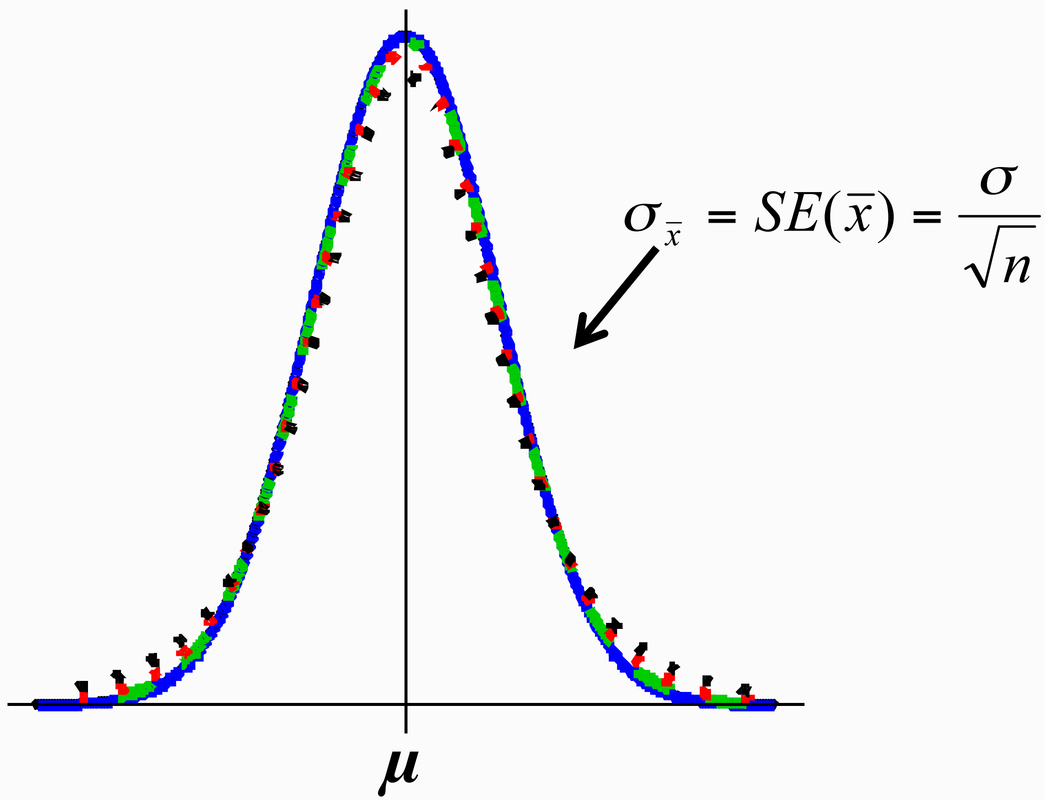
- Technically this is true for “large *n*”: for this course, we’ll say *n* > 60; but when *n* is smaller, sampling distribution is not quite normal, but follows a *t-distribution*



$$\sigma_{\bar{x}} = SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

t-distributions

- The t-distribution is the “fatter, flatter cousin” of the normal: t-distribution is uniquely defined by degrees of freedom



Why the t?

- Basic idea: remember, the true SE($\bar{x}$) is given by the formula

$$\sigma_{\bar{x}} = SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

- But of course we don't know σ, and replace with s to estimate

$$S\hat{E}(\bar{x}) = \frac{s}{\sqrt{n}}$$

- In small samples, there is a lot of sampling variability in s as well: so this estimate is less precise

- To account for this additional uncertainty, we have to go slightly more than $\pm 2 \times S\hat{E}(\bar{x})$ to get 95% coverage under the sampling distribution

Underlying Assumptions

- How much bigger the 2 needs to be depends on the sample size
- You can look up the correct number in a "t-table" or "t-distribution" with *n–1* degrees of freedom

The t-distribution

- So if we have a smaller sample size, we will have to go out more than 2 SEs to achieve 95% confidence
- How many standard errors we need to go depends on the degrees of freedom—this is linked to sample size
- The appropriate degrees of freedom are *n - 1*
- One option: you can look up the correct number in a "t-table" or "t-distribution" with *n - 1* degrees of freedom

$$\bar{x} \pm t_{.95,n-1} \times S\hat{E}(\bar{x}) \Rightarrow$$

$$\bar{x} \pm t_{.95,n-1} \times \frac{s}{\sqrt{n}}$$

Notes on the t-Correction

- The particular t-table gives the number of SEs needed to cut off 95% under the sampling distribution

df	t	df	t
1	12.706	12	2.179
2	4.303	13	2.160
3	3.182	14	2.145
4	2.776	15	2.131
5	2.571	20	2.086
6	2.447	25	2.060
7	2.365	30	2.042
8	2. 360	40	2.021
9	2.262	60	2.000
10	2.228	120	1.980
11	2.201	∞	1.960

Notes on the t-Correction

- You can easily find a t-table for other cutoffs (90%, 99%) in any stats text or by searching the internet
- Also, using the *cii* command takes care of this little detail
- The point is not to spend a lot of time looking up t-values: more important is a basic understanding of why slightly more needs to be added to the sample mean in smaller samples to get a valid 95% CI
- The interpretation of the 95% CI (or any other level) is the same as discussed before

Example

- Small study on response to treatment among 12 patients with hyperlipidemia (high LDL cholesterol) given a treatment

- Change in cholesterol post–pre treatment computed for each of the 12 patients

- Results: $\bar{x}_{change} = -1.4\ mmol/L$

  $s_{change} = 0.55\ mmol/L$

Example

- 95% confidence interval for true mean change

$$\bar{x} \pm t_{.95,11} \times S\hat{E}(\bar{x}) \Rightarrow$$

$$\bar{x} \pm 2.2 \times S\hat{E}(\bar{x}) \Rightarrow$$

$$-1.4 \pm 2.2 \times \frac{0.55}{\sqrt{12}} \Rightarrow$$

$$(-1.75\,, mmol/L, \text{-}\ 1.05\ mmol/L)$$

Using Stata to Create Other CIs for a Mean

- The "cii" command,

```
. cii 12  -1.4 .55

    Variable |        Obs        Mean     Std   . Err.       [95% Conf. Interval]
------------- +-----------------------------------------------------------------
             |         12          -1.4     .1587713         -1.749453    -1.050547
```