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An Introduction to Hypothesis Testing: The Paired t-Test

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Lecture Topics

- Comparing two groups: the paired data situation
- Hypothesis testing: the null and alternative hypotheses
- Relationships between confidence intervals and hypothesis testing when comparing means
- p-values: definition, calculations, and more information



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Section A

The Paired t-Test; the Confidence Interval Component

Comparison of Two Groups

- Are the population means different? (continuous data)
- Paired design
 - Before-after data
 - Twin data
 - Matched case-control
- Two independent sample design
 - Randomized trial
 - Smokers to non-smokers

Paired Design—Example: Before vs. After

- Why pairing?
 - Control extraneous noise
 - Each observation acts as a control
 - Good way to get preliminary data/estimates to be used to develop further research

Paired Design—Example: Before vs. After

- Ten non-pregnant, pre-menopausal women 16-49 years old who were beginning a regimen of oral contraceptive (OC) use had their blood pressures measured prior to starting OC use and three-months after consistent OC use¹
- The goal of this small study was to see what, if any, changes in average blood pressure were associated with OC use in such women
- The data on the following slides shows the resulting pre- and post-OC use systolic BP measurements for the 10 women in the study

¹ Data taken from Rosner B Fundamentals of Biostatistics, 6th ed. (2005) Duxbury Press.

Blood Pressure and Oral Contraceptive Use

	BP Before OC	BP After OC	After-Before
1.	115	128	13
2.	112	115	3
3.	107	106	-1
4.	119	128	9
5.	115	122	7
6.	138	145	7
7.	126	132	6
8.	105	109	4
9.	104	102	-2
10.	115	117	2

$$\bar{x}_{before} = 115.6$$

$$\bar{x}_{after} = 120.4$$

$$\bar{x}_{diff} = 4.8$$

Blood Pressure and Oral Contraceptive Use

- The sample average of the differences is 4.8

- Also note $\bar{x}_{diff} = \bar{x}_{after} - \bar{x}_{before}$

- The sample standard deviation (s) of the differences is $s_{diff} = 4.6$

- Standard deviation of differences found by using the formula:

$$s_{diff} = \sqrt{\frac{\sum_{i=1}^n (x_{diff_i} - \bar{x}_{diff})^2}{n - 1}}$$

- Where:

- ▶ Each x_{diff_i} represents an individual difference
and

- ▶ \bar{x}_{diff} is the mean difference

Blood Pressure and Oral Contraceptive Use

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$$\bar{x}_{before} = 115.6$$

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Note

- In essence, what we have done is reduce the BP information on two samples (women prior to OC use, women after OC use) into one piece of information: information on the differences in BP between the samples
- This is standard protocol for comparing paired samples with a continuous outcome measure

Confidence Interval Approach

- Want to draw a conclusion about a population parameter
 - In a population of women who use oral contraceptives, is the average (expected) change in blood pressure (after-before) 0 or not?
- Sometimes the term *expected* is used for the population average
- μ is the expected (population) mean change in blood pressure
- CI approach allows us to create a range of possible values for μ using data from a single, imperfect (paired) sample

95% Confidence Interval

- 95% confidence interval for mean change in BP in population of women taking oral contraceptives, after starting OC use compared to before OC use

$$\bar{x}_{diff} \pm t_{.95,9} \times \hat{SE}(\bar{x}_{diff})$$

$$\bar{x}_{diff} \pm t_{.95,9} \times \frac{s_{diff}}{\sqrt{10}}$$

$$4.8 \pm 2.26 \times \left(\frac{4.6}{\sqrt{10}} \right)$$

1.5 mmHg to 8.1 mmHg

95% Confidence Interval

- 95% confidence interval for mean change in BP in population of women taking oral contraceptives, after starting OC use compared to before OC use using *cii* in Stata

```
. cii 10 4.8 4.6
```

Variable	Obs	Mean	Std. Err.	[95% Conf. Interval]	
	10	4.8	1.454648	1.509358	8.090642

95% Confidence Interval

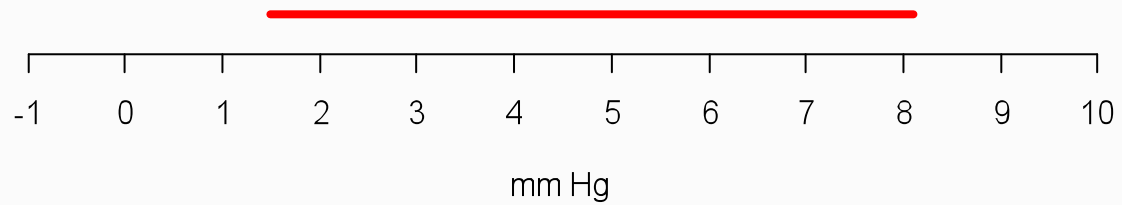
- 95% confidence interval for mean change in BP in population of women taking oral contraceptives, after starting OC use compared to before OC use using *cii* in Stata

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```

Variable	Obs	Mean	Std. Err.	[95% Conf. Interval]	
	10	4.8	1.454648	1.509358	8.090642

Notes

- The number 0 is **NOT** in confidence interval (1.5-8.1)



Notes

- The number 0 is **not** in confidence interval (1.5-8.1)
 - Because 0 is not in the interval, this suggests there is a non-zero change in BP over time
 - The phrase “statistically significant” change is used to indicate a non-zero mean change

Notes

- The BP change could be due to factors other than oral contraceptives
 - Changes in weather over pre- and -post period
 - Changes in personal stress
 - Other changes?
- A control group of comparable women who were not taking oral contraceptives would strengthen this study
 - This is an example of a *pilot study*—a small study done just to generate some evidence of a possible association
 - This can be followed up with a larger, more scientifically rigorous study