JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Section C

The Paired t-Test; Two More Examples

Clinical Agreement by Two Diagnosing Physicians

- Two different physicians assessed the number of palpable lymph nodes in 65 randomly selected male sexual contacts of men with AIDS or AIDS-related conditions[1]

	Doctor 1	Doctor 2	Difference
Mean ($\bar{x}$)	**7.91**	**5.16**	**-2.75**
sd (*s*)	**4.35**	**3.93**	**2.83**

[1]Example based on data taken from Rosner, B. (2005). *Fundamentals of Biostatistics*, sixth. ed. Duxbury Press. (Based on research by Coates, et al. (1988). Assessment of generalized … *Journal of Clinical Epidemiology*, 41(2).

95% Confidence Interval

- 95% CI for difference in mean number of lymph nodes, Doctor 2 compared to Doctor 1

$$\bar{x}_{diff} \pm 2 \times \hat{SE}(\bar{x}_{diff})$$

$$\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{65}}$$

$$2.75 \pm 2 \times \left(\frac{2.83}{\sqrt{65}} \right)$$

$$-3.45 \ \ to \ \ -2.05$$

Getting a p-Value

- Hypotheses
  - H_o: $\mu_{diff} = 0$
  - H_A: $\mu_{diff} \neq 0$

- First, start by "assuming" null is true and computing distance (in SEs) between $\bar{x}_{diff}$ and 0
  - Sample result is 7.8 SEs below 0—*is this unusual?*

$$t = \frac{\bar{x}_{diff} - 0}{S\hat{E}(\bar{x})} = \frac{-2.75}{2.83/\sqrt{65}} = -7.8$$

Getting a p-Value

- Sample result is 7.8 SEs below 0—*is this unusual?*
  - See where this falls on sampling distribution of all possible mean differences based on random samples of 65 patients
    - Theory tells us this is normal

- The p-value is probability of being 7.8 or more standard errors from 0 under a standard normal curve
  - Without looking up, we know p <<< .001!

Everything with Stata

- ttesti 65 -2.75 2.83 0

```
. ttesti  65 -2.75 2.83 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      65       -2.75    .3510183        2.83    -3.45124    -2.04876
------------------------------------------------------------------------------
    mean = mean(x)                                                t =  -7.8343
Ho: mean = 0                                     degrees of freedom =       64

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 1.0000
```

Oat Bran and LDL Cholesterol

- Cereal and cholesterol: 14 males with high cholesterol given oat bran cereal as part of diet for two weeks, and corn flakes cereal as part of diet for two weeks

	Corn Flakes	Oat Bran	Difference
Mean ($\bar{x}$)	**4.44 mmol/dL**	**4.08**	**0.36**
sd (*s*)	**1.0**	**1.1**	**0.40**

[1]Example based on data taken from Pagano, M. (2000). *Principles of Biostatistics*, 2nd ed. Duxbury Press. Based on research by Anderson J, et al. (1990). Oat Bran Cereal Lowers … *American Journal of Clinical Nutrition*, 52.

95% Confidence Interval

- 95% CI for difference in mean LDL, corn flakes vs. oat bran

$$\bar{x}_{diff} \pm t_{.95,13} \times S\hat{E}(\bar{x}_{diff})$$

$$\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{14}}$$

$$0.36 \pm 2 \times \left(\frac{.040}{\sqrt{14}} \right)$$

$$0.13 \;\; to \;\; 0.60 \; mmol/dL$$

Getting a p-Value

- Hypotheses
  - H_o: $\mu_{diff} = 0$
  - H_A: $\mu_{diff} \neq 0$

- First, start by “assuming” null is true, and computing distance (in SEs) between $\bar{x}_{diff}$ and 0
  - Sample result is 3.3 SEs above 0—*is this unusual?*

$$t = \frac{\bar{x}_{diff} - 0}{\hat{SE}(\bar{x})} = \frac{.036}{.04/\sqrt{14}} \approx 3.3$$

Getting a p-Value

- Sample result is 3.3 SEs above 0—*is this unusual?*
  - See where this falls on sampling distribution of all possible mean differences based on random samples of 14 patients: theory tells us this is t_{13}

- The p-value is probability of being 3.3 or more standard errors from 0 under a t_{13} curve: look up in table or go to Stata

Everything with Stata

- cii 14 .36 .40 0

```
. ttesti 14 .36 .40 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      14         .36    .1069045          .4    .1290469    .5909531
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   3.3675
Ho: mean = 0                                     degrees of freedom =       13

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.9975         Pr(|T| > |t|) = 0.0050          Pr(T > t) = 0.0025
```

Direction of Comparison is Arbitrary

- Does not impact overall results at all, direction changes, so signs of mean diff and CI endpoints change; but message exactly the same

```
. ttesti 14 -.36 .40 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      14        -.36    .1069045          .4   -.5909531   -.1290469
------------------------------------------------------------------------------
    mean = mean(x)                                                t =  -3.3675
Ho: mean = 0                                     degrees of freedom =       13

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.0025         Pr(|T| > |t|) = 0.0050          Pr(T > t) = 0.9975
```

Summary: Paired t-Test

- Designate null and alternative hypotheses

- Collect data

- Compute difference in outcome for each paired set of observations
  - Compute $\bar{x}_{diff}$, sample mean of the paired differences
  - Compute *s*, sample standard deviation of the differences

Summary: Paired t-Test

- Compute 95% (or other level) CI for true mean difference between paired groups compared
  - "Big *n*" ($n > 60$)

$$\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{n}}$$

  - "Small *n*" ($n \leq 60$)

$$\bar{x}_{diff} \pm t_{.95,n-1} \times \frac{s_{diff}}{\sqrt{n}}$$

Summary: Paired t-Test

- To get p-values
  - Start by assuming H_o true
  - Measure distance of sample result from μ_o

$$t = \frac{\bar{x}_{diff} - \mu_o}{\hat{SE}(\bar{x}_{diff})}$$

  - Usually, μ_o=0, so:

$$t = \frac{\bar{x}_{diff}}{\hat{SE}(\bar{x}_{diff})} = \frac{\bar{x}_{diff}}{s_{diff} / \sqrt{n}}$$

Summary: Paired t-Test

- Compare test statistics (distance) to appropriate distribution to get p-value
  - Reminder: p-value measures how likely your sample result (and other result less likely) are if null is true

Summary: Paired t-Test/Paired Data Situations

- Example 1
  - The blood pressure/OC example
- Example 2
  - Degree of clinical agreement, each patient received two assessments
- Example 3
  - Single group of men given two different diets at in two different time periods
  - LDL cholesterol levels measured at end of each diet

Summary: Paired t-Test/Paired Data Situations

- Twin study

- Matched case control scenario
  - Suppose we wish to compare levels of a certain biomarker in patients with a given disease versus those without