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Section D

The p-Value *in Even More Detail!*

p-Values

- p-values are probabilities (numbers between 0 and 1)

- Small p-values mean that the sample results are unlikely when the null is true

- The p-value is the probability of obtaining a result as extreme or more extreme than you did by chance alone assuming the null hypothesis H_0 is true
  - How likely your sample result (and other result less likely) are if null is true

p-Values

- The p-value is not the probability that the null hypothesis is true!

- The p-value alone imparts no information about scientific/ substantive content in result of a study

- Example: from Example 3, the researchers found a statistically significant ($p=0.005$!) difference in average LDL cholesterol levels in men who had been on a diet including corn flakes versus the same men on a diet including oat bran cereal
  - Which diet showed lower average LDL levels?
  - How much was the difference; does it mean anything nutritionally?

p-Values

- If the p-value is small either a very rare event occurred and
  - H_0 is true

    or

  - H_0 is false

- Type I error
  - Claim H_A is true when in fact H_0 is true
  - The probability of making a Type I error is called the *alpha-level (α-level)* or *significance level*

Note on the p-Value and the Alpha-Level

- If the p-value is less then some pre-determined cutoff (e.g., .05), the result is called *statistically significant*

- This cutoff is the α*-level*
  - α-level is the probability of a type I error
  - It is the probability of falsely rejecting H_0 when H_0 true

- Idea: to keep the chance of “making a mistake” when the H_0 is true low and only reject if the sample result is “unlikely”
  - Unlikeliness threshold is determined by α*-level*

Note on the p-Value and the Alpha-Level

- Truth versus decision made by hypothesis testing

	TRUTH	
	H_o	H_A
Reject H_o	**Type I Error** **alpha-level**	**Power** **1-beta**
Not Reject H_o		**Type II Error** **beta**

Note on the p-Value and the Alpha-Level

- Truth versus decision made by hypothesis testing

	TRUTH: H_o	TRUTH: H_A
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Note on the p-Value and the Alpha-Level

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Note on the p-Value and the Alpha-Level

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More on p-Value: One-Sided vs. Two-Sided Controversy

- Two-sided p-value (BP/OC: p = .009)
  - Probability of a result as or more extreme than observed (either positive or negative)

- One-sided p-value
  - Probability of a more extreme positive result than observed or a more extreme negative result: only considers extremes in one direction of null when evaluation how likely your sample result is (and results less likely)
  - If the direction of the alternative hypothesis in the one-sided test is the same as the direction of the sample result in terms of above/below the null, then the one-sided p-value with be half the two-sided p-value

Stata Output

- One-sided alternative: true mean difference >0
  - Sample mean difference was greater than 0

```
. ttesti 10 4.8 4.6 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      10         4.8    1.454648         4.6    1.509358    8.090642
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   3.2998
Ho: mean = 0                                     degrees of freedom =        9

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.9954         Pr(|T| > |t|) = 0.0092          Pr(T > t) = 0.0046
```

Stata Output

- One-sided alternative: true mean difference <0
  - Sample mean difference was greater than 0

```
. ttesti 10 4.8 4.6 0

One-sample t test
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More on the p-Value

- In some cases, a one-sided alternative may not make scientific sense
  - In the absence of pre-existing information, in evaluating the BP/OC relationship, wouldn't either result be interesting and useful? (i.e., negative or positive association?)

- In some cases, a one-sided alternative often makes scientific sense
  - For example: not really interested if new treatment is worse than old treatment—only care whether it's better

- However: because of "culture of p-value" and sanctity of ".05," one-sided p-values are viewed with suspicion

- In this course, we will use two-sided p-values exclusively

Connection: Hypothesis Testing and CIs

- The confidence interval gives plausible values for the population parameter
  - "Data take me to the truth"

- Hypothesis testing postulates two choice for the population parameter
  - "Here are two possibilities for the truth; data help me choose one"

95% Confidence Interval

- If 0 is not in the 95% CI, then we would reject H_0 that $\mu = 0$ at level $\alpha = .05$ (the p-value < .05)

- Why?

- With confidence interval we start at sample mean difference and go two standard errors in either direction (or slightly more in small samples)

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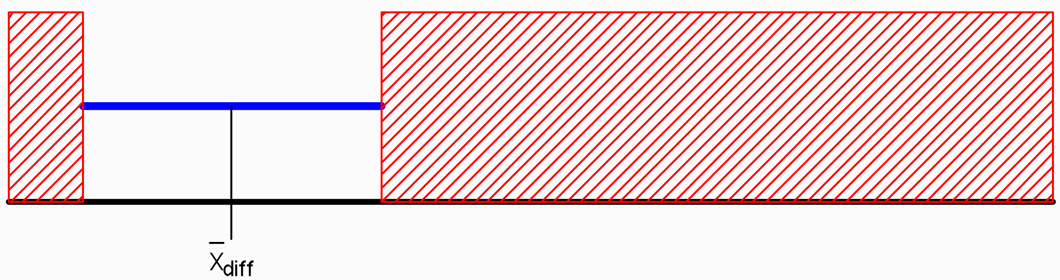
95% Confidence Interval

- If 0 is not in the 95% CI, then this must mean $\bar{x}$ is > 2 standard errors away from 0 (either above or below)

- Hence, the distance (t) will be > 2 or < -2: and the resulting p-value <.05



95% Confidence Interval and p-Value

- In the BP/OC example, the 95% confidence interval tells us that the p-value is less than .05, but it doesn't tell us that it is p = .009

- The confidence interval and the p-value are complementary

- However, you can't get the exact p-value from just looking at a confidence interval, and you can't get a sense of the scientific/ substantive significance of your study results by looking at a p-value

More on the p-Value

- Statistical significance does not imply/prove causation
- For example: in the blood pressure/oral contraceptives example, there could be other factors that could explain the change in blood pressure
- A significant p-value is only ruling out random sampling (chance) as the explanation
- Need a comparison group to better establish causality
  - Self-selected (may be okay)
  - Randomized (better)

More on the p-Value

- *Statistical significance* is not the same as *scientific significance*

- Hypothetical example: blood pressure and oral contraceptives:
  - Suppose:
    - n = 100,000; $\overline{x}_{diff}$ = .03 mmHg; *s* = 4.6 mmHg
    - p-value = .04

- Big *n* can sometimes produce a small p-value, even though the magnitude of the effect is very small (not scientifically/ substantively significant)

- Very important
  - Always report a confidence interval
  - 95% CI: 0.002–0.058 mmHg

More on the p-Value

- *Lack of statistical significance* is not the same as *lack of scientific significance*
  - Must evaluate in context of study, sample size

- Small *n* can sometimes produce a non-significant even though the magnitude of the association at the population level is real and important (our study just can't detect it)

- *Low power* in small sample studies makes not rejecting hard to interpret

- Sometimes small studies are designed without power in mind just to generate preliminary data