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JOHNS HOPKINS
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Section B

Two Sample t-test: Getting a p-value

Hypothesis Test to Compare Two Independent Groups

- Two sample (unpaired) t-test

- Is the (mean) weight change equal in the two diet groups?
  - H_o: $\mu_1 = \mu_2$
  - H_A: $\mu_1 \neq \mu_2$

- In other words, is the expected difference in weight change zero?
  - H_o: $\mu_1 - \mu_2 = 0$
  - H_A: $\mu_1 - \mu_2 \neq 0$

Hypothesis Test to Compare Two Independent Groups

- Recall, general "recipe" for hypothesis testing . . .
  1. Start by assuming H_o true
  2. Measure distance of sample result from μ_o (here again its 0)
  3. Compare test statistic (distance) to appropriate distribution to get p-value

$$t = \frac{(observed\ dif\ f) - (null\ dif\ f}{SE\ of\ observed\ dif\ f\ erenc}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\hat{E}(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}}$$

Diet Type and Weight Loss Study

- In the diet types and weight loss study, recall:

$$\bar{x}_1 - \bar{x}_2 = -3.9kg$$

$$\hat{SE}(\bar{x}_1 - \bar{x}_2) = 1.17kg$$

- So in this study:

$$t = \frac{-3.9}{1.17} \approx -3.3$$

  - So this study result was 3.3 standard errors below the null mean of 0 (i.e., 3.3 standard errors from the mean weight less expected if null was true)

How Are p-values Calculated?

- Is a result 3.3 standard errors below 0 unusual?
  - It depends on what kind of distribution we are dealing with

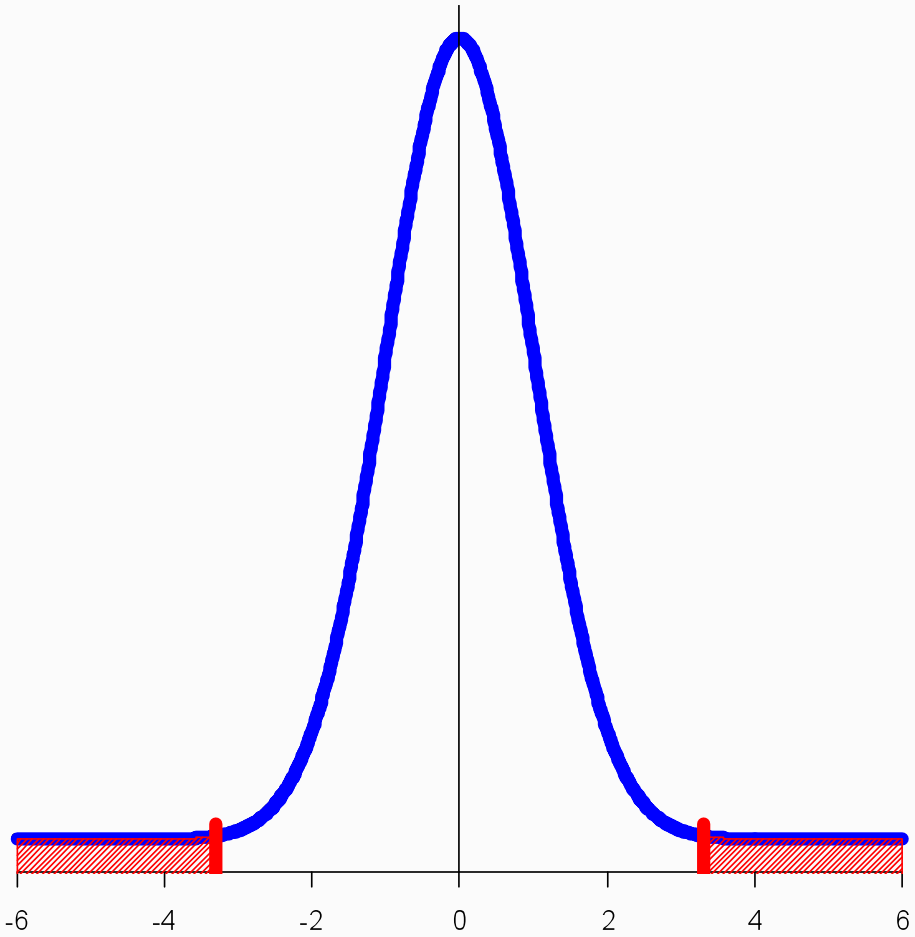
- The p-value is the probability of getting a test statistic as extreme as (or more extreme than) what you observed (-3.3) by chance if was true

- The p-value comes from the sampling distribution of the difference in two sample means

- What is the sampling distribution of the difference in sample means?
  - If both groups are large (more than 60 subjects) then this distribution is approximately normal
  - This sampling distribution will be centered at true difference
  - Under null hypothesis, this true difference is 0

Diet/Weight Loss Sample

- To compute a p-value, we would need to compute the probability of being 3.3 or more standard errors away from 0 on a standard normal curve



How to Use Stata to Perform a 2-Sample T-Test

- Command syntax:
  - ttesti n_1 $\bar{x}_1$ s_1 n_2 $\bar{x}_2$ s_2 , unequal

```
. ttesti 64 -5.7 8.6 68 -1.8 3.9, unequal

Two-sample t test with unequal variances
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      64        -5.7       1.075         8.6   -7.848216   -3.551784
       y |      68        -1.8     .4729445         3.9   -2.744001   -.8559989
---------+--------------------------------------------------------------------
combined |     132   -3.690909     .5978226    6.868458   -4.873545   -2.508273
---------+--------------------------------------------------------------------
    diff |              -3.9     1.174437               -6.234436   -1.565564
------------------------------------------------------------------------------
    diff = mean(x) - mean(y)                                      t =  -3.3207
Ho: diff = 0                     Satterthwaite's degrees of freedom =  86.6941

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0007         Pr(|T| > |t|) = 0.0013          Pr(T > t) = 0.9993
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Summary: Weight Loss Example

- **Statistical method**
  - "We randomly assigned 132 severely obese patients . . . to a carbohydrate restricted (low-carbohydrate) diet or a calorie- and fat-restricted diet"
  - "For comparison of continuous variables between the two groups, we calculated the change from baseline to six months in each subject, and compared the mean changes in the two diet groups using an unpaired t-test"

- **Result**
  - "Subjects on the low-carbohydrate diet lost more weight than those on a low fat diet (95% confidence interval for the difference in weight loss between groups, -1.6 to -6.2 kg; $p < .01$)"