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Section G

Comparing Means between More than Two Independent Populations
Motivating Example

- Suppose you are interested in the relationship between smoking and mid-expiratory flow (FEF), a measure of pulmonary health.

- Suppose you recruit study subjects and classify them into one of six smoking categories:
  - Nonsmokers (NS)
  - Passive smokers (PS)
  - Non-inhaling smokers (NI)
  - Light smokers (LS)
  - Moderate smokers (MS)
  - Heavy smokers (HS)
Motivating Example

- You are interested in whether differences exist in mean FEF amongst the six groups

- Main outcome variable is mid-expiratory flow (FEF) in liters per second
Motivating Example

- One strategy is to perform lots of two-sample t-tests (for each possible two-group comparison)

- In this example, there would be 15 comparisons you would need to do!
  - NS to PS, NS to NI, and so on . . .
Motivating Example

- It would be nice to have one “catch-all” test
  - Something which would tell you whether there were any differences amongst the six groups
  - If so, you could then do group to group comparisons to look for specific group differences
Extension of the Two-Sample t-Test

- Analysis of variance (One-Way ANOVA)
  - The t-test compares means in two populations
  - ANOVA compares means amongst more than two populations with one test

- The p-value from ANOVA helps answer the question
  - “Are there any differences in the means among the populations?”
Extension of the Two-Sample t-Test

- General idea behind ANOVA, comparing means for k-groups (k > 2):
  - $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$
  - $H_A : \text{At least one mean different}$
Smoking and FEF (Forced Mid-Expiratory Flow Rate)*

A sample of over 3,000 persons was classified into one of six smoking categorizations based on responses to smoking related questions.

Example 1

- Nonsmokers (NS)
- Passive smokers (PS)
- Non-inhaling smokers (NI)
- Light smokers (LS)
- Moderate smokers (MS)
- Heavy smokers (HS)
Smoking and FEF

- From each smoking group, a random sample of 200 men was drawn (except for the non-inhalers, as there were only 50 male non-inhalers in the entire sample of 3,000)
- FEF measurements were taken on each of the subjects
Example 1—Table

Data summary

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean FEF (L/s)</th>
<th>SD FEF (L/s)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS</td>
<td>3.78</td>
<td>0.79</td>
<td>200</td>
</tr>
<tr>
<td>PS</td>
<td>3.30</td>
<td>0.77</td>
<td>200</td>
</tr>
<tr>
<td>NI</td>
<td>3.32</td>
<td>0.86</td>
<td>50</td>
</tr>
<tr>
<td>LS</td>
<td>3.23</td>
<td>0.78</td>
<td>200</td>
</tr>
<tr>
<td>MS</td>
<td>2.73</td>
<td>0.81</td>
<td>200</td>
</tr>
<tr>
<td>HS</td>
<td>2.59</td>
<td>0.82</td>
<td>200</td>
</tr>
</tbody>
</table>

Based on a one-way analysis of variance, there are statistically significant differences in FEF levels among the six smoking groups (p < .001)
What’s the Rationale behind Analysis of Variance?

- The variation in the sample means between groups is compared to the variation within a group.

- If the between group variation is a lot bigger than the within group variation, that suggests there are some differences among the populations.
Summary: Smoking and FEF

- **Statistical methods**
  - 200 men were randomly selected from each of five smoking classification groups (non-smoker, passive smokers, light smokers, moderate smokers, and heavy smokers), as well as 50 men classified as non-inhaling smokers for a study designed to analyze the relationship between smoking and respiratory function.
Summary: Smoking and FEF

- **Statistical Methods**
  - Analysis of variance was used to test for any differences in FEF levels amongst the six groups of men.
  - Individual group comparisons were performed with a series of two sample t-tests, and 95% confidence intervals were constructed for the mean difference in FEF between each combination of groups.
  - Analysis of variance showed statistically significant (p < .001) differences in FEF between the six groups of smokers.
  - Non-smokers had the highest mean FEF value, 3.78 L/s, and this was statistically significantly larger than the five other smoking-classification groups.
Summary: Smoking and FEF

- **Results**
  - Analysis of variance showed statistically significant (p < .001) differences in FEF between the six groups of smokers.
  - Non-smokers had the highest mean FEF value, 3.78 L/s, and this was statistically significantly larger than the five other smoking-classification groups.
  - The mean FEF value for non-smokers was 1.19 L/s higher than the mean FEF for heavy smokers (95% CI 1.03-1.35 L/s), the largest mean difference between any two smoking groups.
  - Confidence intervals for all smoking group FEF comparisons are in Table 1.
Example 2

- **FEV1 and three medical centers**
  - Data was collected on 63 patients with coronary artery disease at 3 different medical centers (Johns Hopkins, Ranchos Los Amigos Medical Center, St. Louis University School of Medicine)
  - Purpose of study to investigate effects of carbon monoxide exposure on these patients
  - Prior to analyzing CO effects data, researchers wished to compare the respiratory health of these patients across the three medical centers

Example 2

- Snippet of data in Stata

<table>
<thead>
<tr>
<th>center</th>
<th>fev1</th>
</tr>
</thead>
<tbody>
<tr>
<td>JH</td>
<td>2.63</td>
</tr>
<tr>
<td>JH</td>
<td>2.53</td>
</tr>
<tr>
<td>RLA</td>
<td>3.22</td>
</tr>
<tr>
<td>RLA</td>
<td>2.88</td>
</tr>
<tr>
<td>RLA</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>RLA</td>
<td>2.89</td>
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<tr>
<td>RLA</td>
<td>3.77</td>
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<tr>
<td>RLA</td>
<td>3.29</td>
</tr>
<tr>
<td>RLA</td>
<td>3.39</td>
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<td></td>
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<tr>
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<td>2.64</td>
</tr>
</tbody>
</table>
Boxplots

- FEV1 values by center
Example 2

- **ANOVA with Stata**
  - Syntax: `oneway outcome_var group_var`

```stata
donway fev1 center

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>1.58283723</td>
<td>2</td>
<td>.791418613</td>
<td>3.12</td>
<td>0.0520</td>
</tr>
<tr>
<td>Within groups</td>
<td>14.4802561</td>
<td>57</td>
<td>.254039581</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>16.0630933</td>
<td>59</td>
<td>.272255819</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bartlett's test for equal variances: chi2(2) = 0.0583  Prob>chi2 = 0.971
Example 2

- **ANOVA with Stata**
  
  ```
syntax oneway outcome_var group_var

oneway fev1 center
  
  Analysis of Variance
  
<table>
<thead>
<tr>
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</table>

Bartlett's test for equal variances: chi2(2) = 0.0583 Prob>chi2 = 0.971
  ```
Example 2

- FEV and 3 medical centers 95% CIs for FEV1 by medical center

```stata
>bys center: ci fev1
```

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Err.</th>
<th>[ 95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>fev1</td>
<td>21</td>
<td>2.62619</td>
<td>.1082732</td>
<td>2.400337  2.852044</td>
</tr>
<tr>
<td>fev1</td>
<td>16</td>
<td>3.0325</td>
<td>.13081</td>
<td>2.753685  3.311315</td>
</tr>
<tr>
<td>fev1</td>
<td>23</td>
<td>2.878696</td>
<td>.1037809</td>
<td>2.663467  3.093924</td>
</tr>
</tbody>
</table>
```