JOHNS HOPKINS
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SCHOOL of PUBLIC HEALTH

Section B

Two Sample z-test: Getting a p-value

Hypothesis Test to Compare Two Proportions

- Two sample z-test

- Are the proportions of infants contracting HIV within 18 months-of-birth equivalent at the population level for those whose mothers are treated with AZT versus untreated (placebo)?
  - H_o: $p_1 = p_2$
  - H_A: $p_1 \neq p_2$

- In other words, is the expected difference in proportions zero?
  - H_o: $p_1 - p_2 = 0$
  - H_A: $p_1 - p_2 \neq 0$

Hypothesis Test to Compare Two Independent Groups

- Recall, general "recipe" for hypothesis testing . . .
  1. Start by assuming H_o true
  2. Measure distance of sample result from μ_o (here again its 0)
  3. Compare test statistic (distance) to appropriate distribution to get p-value

$$z = \frac{(observed\ dif\ f) - (null\ dif\ f}{SE\ of\ observed\ dif\ f\ erenc}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{S\hat{E}(\hat{p}_1 - \hat{p}_2)} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1 \times (1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 \times (1 - \hat{p}_2)}{n_2}}}$$

Infant HIV/ AZT Study

- In the infant HIV/AZT study, recall:

$$\hat{p}_1 - \hat{p}_2 = -0.15$$

$$\hat{SE}(\hat{p}_1 - \hat{p}_2) = 0.036$$

- So in this study:

$$z = \frac{-0.15}{0.36} \approx -4.2$$

  - So this study result was 4.2 standard errors below the null mean of 0 (i.e., 4.2 standard errors from the difference in the proportion of HIV+ infants between the AZT and placebo groups expected if null was true)

How Are p-values Calculated?

- Is a result 4.2 standard errors below 0 unusual?
  - It depends on what kind of distribution we are dealing with

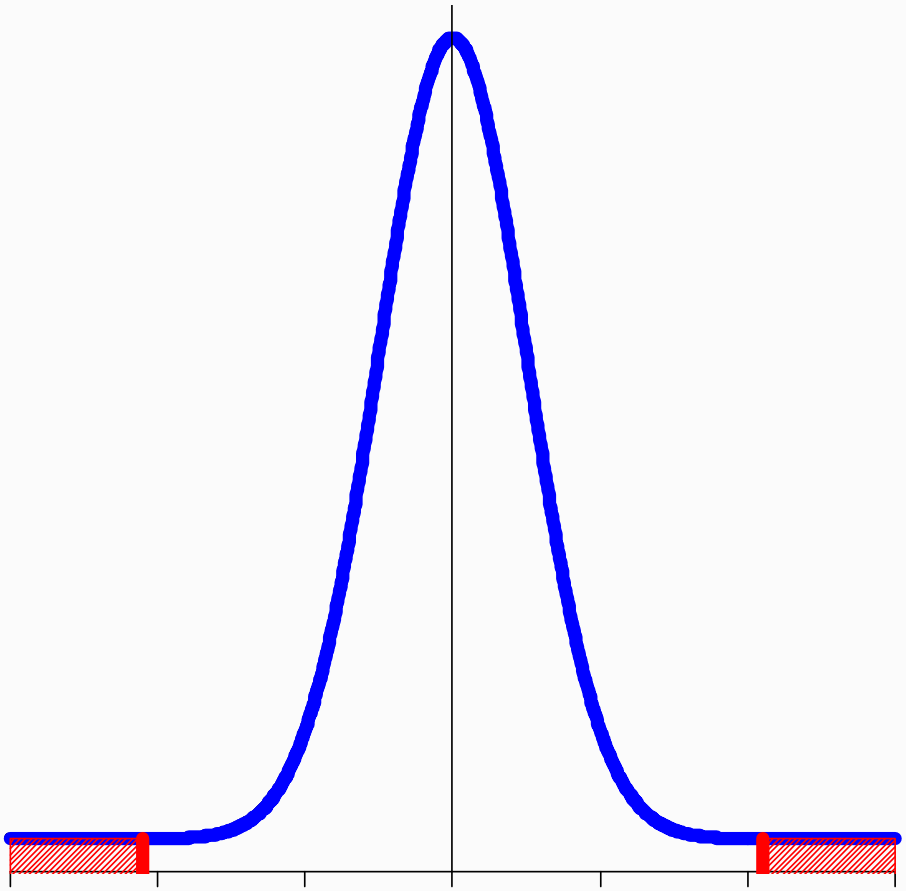
- The p-value is the probability of getting a test statistic as (or more extreme than) what you observed (-4.2) by chance

- The p-value comes from the sampling distribution of the difference in two sample proportions

- What is the sampling distribution of the difference in sample means?
  - If both groups are large then this distribution is approximately normal
  - This sampling distribution will be centered at true difference,
  - Under null hypothesis, this true difference is 0

Diet/Weight Loss Sample

- To compute a p-value, we would need to compute the probability of being 4.2 or more standard errors away from 0 on a standard normal curve



AZT Study

- If we were to look this up on a normal table, we would find a very low p-value ($p < .001$)

- This method is also essentially equivalent to the chi-square ($\chi 2$) method
  - Gives about the same answer (p-value)
  - This is how Stata approaches it
  - We will discuss chi-square method in more detail shortly: for now, just "take on faith" that it is equivalent so we can show you how to get the p-value, 95% CI (etc.) using Stata

To Do in Stata: Display Data in a 2x2 Table

- Stata "thinks" of data in a 2x2 (contingency) table
- Two rows and two columns

		Drug Group: AZT	Drug Group: Placebo	
HIV Transmission	Yes	13	40	53
	No	167	143	310
		180	183	363

To Do in Stata: Display Data in a 2x2 Table

- We can get Stata to give us a 95% CI for the difference in proportions, and a p-value by using the csi command

- Syntax *csi a b c d*
  - Based a 2x2 table using our sample results as such

		Exposure	
		Yes	No
Outcome	Yes	a	b
	No	c	d

Using Stata: AZT/HIV Example

- *csi 13 40 167 143*

		Drug Group: AZT	Drug Group: Placebo	
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	No	167	143	310
		180	183	363

Using Stata: AZT/HIV Example

- Results from *csi* command

```
.   csi 13 40 167 143

                 |   Exposed   Unexposed  |      Total
-----------------+------------------------+-----------
           Cases |        13          40  |         53
        Noncases |       167         143  |        310
-----------------+------------------------+-----------
           Total |       180         183  |        363
                 |                        |
            Risk |   .0722222    .2185792  |   .1460055
                 |                        |
                 |      Point estimate    |  [95% Conf. Interval]
                 |------------------------+----------------------
 Risk difference |         -.146357       | -.2171766   -.0755374
      Risk ratio |         .3304167       |  .1829884    .5966235
 Prev. frac. ex. |         .6695833       |  .4033765    .8170116
 Prev. frac. pop |         .3320248       |
                 +-------------------------------------------------
                           chi2(1) =    15.59  Pr>chi2 = 0.0001
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Using Stata: AZT/HIV Example

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Summary: AZT Study

- **Statistical method**
  - "We conducted a randomized, double-blind, placebo-controlled trial of the efficacy and safety of zidovudine (AZT) in reducing the risk of maternal-infant HIV transmission"

Summary: AZT Study

- **Statistical method**
  - The proportion of infants diagnosed as HIV positive within 18 months of birth was compared between the AZT and placebo groups using a two-sample z-test of proportions
  - 95% confidence intervals were computed for the 18-month infection proportion in each group and for the difference in proportions between both groups

Summary: AZT Study

- **Results**
  - The proportion of infants who tested positive for HIV within 18 months of birth was seven percent (95% CI 4 -12%) in the AZT group and twenty-two percent in the placebo group (95% CI 16 – 28%)
  - This difference is statistically significant ($p < .001$)

Summary: AZT Study

- **Results**
  - The study results estimate the decrease in the proportion of HIV positive infants born to HIV positive mothers associated with AZT to be as low as 8% and as high as 22%