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## Section G

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FYI: Sampling Behavior of Relative Risks/Odds Ratios

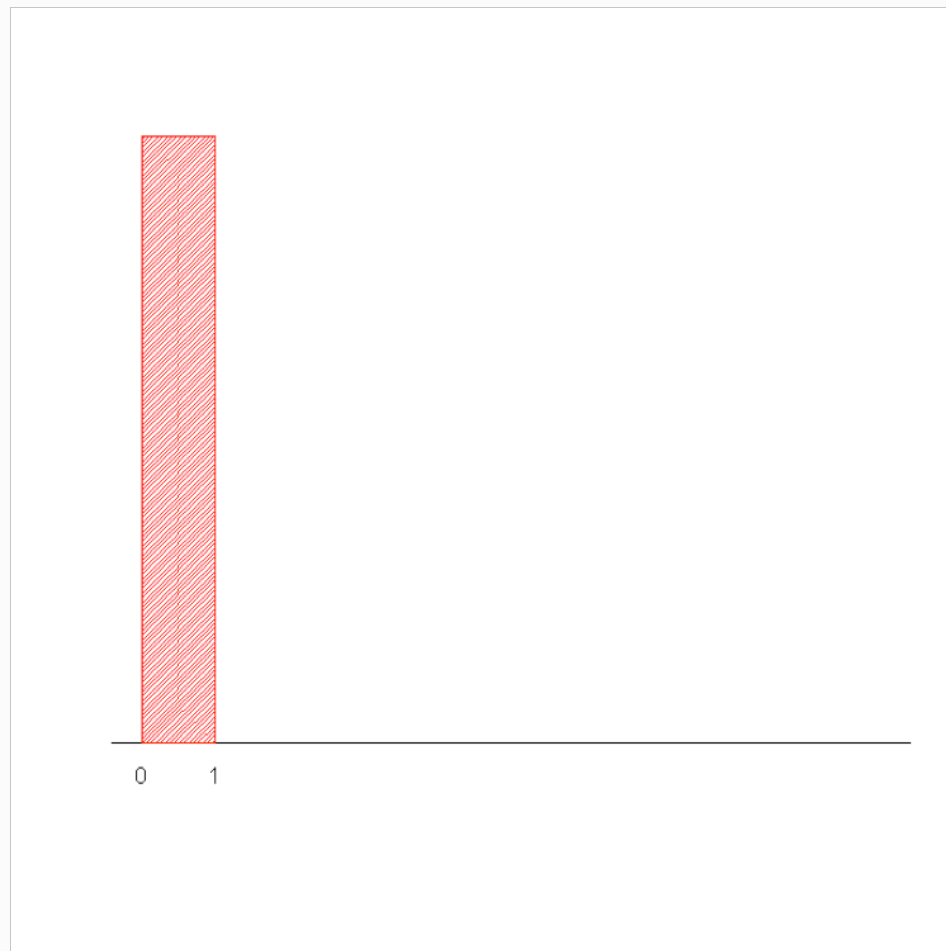
# Sampling Behavior of Ratios Is Not Normal

- The sampling behavior of ratios (like the RR, OR) can be quite skewed
  - The range of possible values for “positive” and “negative” associations are very different



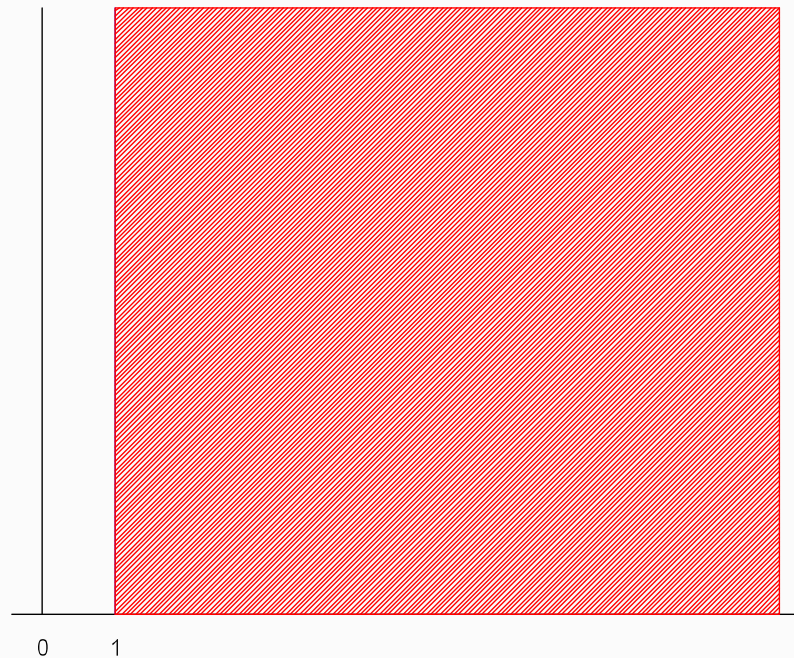
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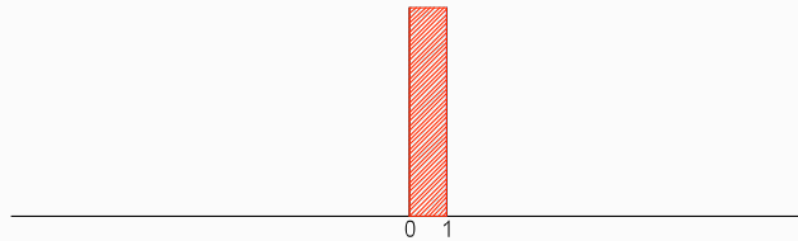
# Sampling Behavior of Ratios Is Not Normal

- The sampling behavior of ratios (like the RR, OR) can be quite skewed
  - The range of possible values for “positive” associations



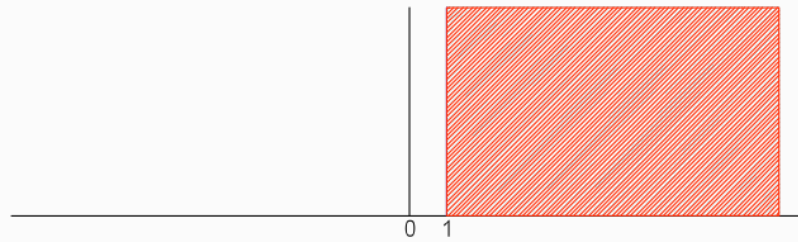
# Sampling Behavior of Ratios is Not Normal

- The ranges are equal on the  $\ln(\text{Ratio})$  scale



# Sampling Behavior of Ratios is Not Normal

- The ranges are equal on the  $\ln(\text{Ratio})$  scale



# Sampling Behavior of Ratios is Not Normal

- Recall standard 2x2 table setup

		Exposure	
		Yes	No
Outcome	Yes	a	b
	No	c	d



## Estimating CI for RR by Hand

- In ratios and standard errors

$$\ln(R\hat{R}) = \ln\left(\frac{\hat{p}_1}{\hat{p}_2}\right)$$

- Standard error, using counts from 2x2 table

$$S\hat{E}(\ln(R\hat{R})) = \sqrt{\frac{c}{a \times n_1} + \frac{d}{b \times n_2}}$$

- 95% CI for  $\ln(RR)$

$$\ln(R\hat{R}) \pm 2 \times S\hat{E}(\ln(R\hat{R}))$$

- To get 95% CI for RR, exponentiate endpoints of above

# HIV/AZT Example

- HIV/mother-infant transmission example

		Drug Group		
		AZT	Placebo	
HIV Transmission	Yes	13	40	53
	No	167	143	310
		180	183	363

# Sampling Behavior of Ratios Is Not Normal

- In ratios and standard errors

$$\ln(R\hat{R}) = \ln\left(\frac{\hat{P}_{AZT}}{\hat{P}_{Placebo}}\right) = \ln\left(\frac{.07}{.22}\right) = \ln(0.33) = -1.11$$

- Standard error, using counts from 2x2 table

$$S\hat{E}(\ln(R\hat{R})) = \sqrt{\frac{167}{13 \times 180} + \frac{143}{40 \times 183}} \approx .30$$

- 95% CI for  $\ln(RR)$

$$-1.11 \pm 2 \times .30 \rightarrow (-1.71, -0.51)$$

- To get 95% CI for RR, exponentiate endpoints of above

$$(e^{-1.71}, e^{-0.51}) \approx (0.18, 0.60)$$

# HIV/AZT Example

- 95% CI from Stata

```
. csi 13 40 167 143, or
```

	Exposed	Unexposed	Total
Cases	13	40	53
Noncases	167	143	310
Total	180	183	363
Risk	.0722222	.2185792	.1460055
	Point estimate	[95% Conf. Interval]	
Risk difference	-.146357	-.2171766	-.0755374
Risk ratio	.3304167	.1829884	.5966235
Prev. frac. ex.	.6695833	.4033765	.8170116
Prev. frac. pop	.3320248		
Odds ratio	.2782934	.1445784	.5363045 (Cornfield)
+-----+-----+-----+-----+			
	chi2(1) =	15.59	Pr>chi2 = 0.0001

# Sampling Behavior of Ratios Is Not Normal

- In ratios and standard errors

$$\ln(OR\hat{R}) = \ln\left(\frac{\hat{p}_1/(1-\hat{p}_1)}{\hat{p}_2/(1-\hat{p}_2)}\right)$$

- Standard error, using counts from 2x2 table

$$S\hat{E}(\ln(OR\hat{R})) = \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$

- 95% CI for  $\ln(OR\hat{R})$

$$\ln(OR\hat{R}) \pm 2 \times S\hat{E}(\ln(OR\hat{R}))$$

- To get 95% CI for OR, exponentiate endpoints of above

# Sampling Behavior of Ratios is Not Normal

- HIV/AZT transmission example

$$\ln(O\hat{R}) = \ln\left(\frac{.07/.93}{.22/.78}\right) \approx \ln(.28) = -1.27$$

- Standard error, using counts from 2x2 table

$$SE(\ln(O\hat{R})) = \sqrt{\frac{1}{13} + \frac{1}{40} + \frac{1}{167} + \frac{1}{143}} \approx .34$$

- 95% CI for  $\ln(O\hat{R})$

$$-1.27 \pm 2 \times .34 \rightarrow (-1.96, -0.60)$$

- To get 95% CI for OR, exponentiate endpoints of above

$$(e^{-1.96}, e^{-0.60}) \approx (0.14, 0.55)$$

# HIV/AZT Example

- 95% CI from Stata

```
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```

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