Statistics for laboratory scientists

Homework problems for lecture 6

1. Consider two events, A and B.
   
   a. Suppose that A and B are *mutually exclusive*. Calculate, in terms of \( \Pr(A) \) and \( \Pr(B) \),
      
      i. \( \Pr(A \text{ or } B) \)
      
      ii. \( \Pr(A \text{ and } B) \)
   
   b. Suppose that A and B are *independent*. Calculate, in terms of \( \Pr(A) \) and \( \Pr(B) \),
      
      i. \( \Pr(A \text{ and } B) \)
      
      ii. \( \Pr(A \text{ or } B) \)
   
   c. Suppose that A and B are *both* mutually exclusive and independent. What can we say about \( \Pr(A \text{ or } B) \), \( \Pr(A \text{ and } B) \), \( \Pr(A) \) and \( \Pr(B) \).

2. Consider a rare recessive trait. Let \( p \) be the frequency of the disease allele, d. (Let + denote the normal allele. Assume Hardy-Weinberg equilibrium and random mating, so that the frequencies of the three genotypes, ++, +d, and dd are \((1-\p)^2\), \(2\p(1-\p)\), and \(\p^2\), respectively.

   Consider picking a woman at random from the population. Let \( U = \{\text{She is unaffected}\} \), \( C = \{\text{She is a carrier}\} \), \( A = \{\text{her brother is affected and her parents are both unaffected}\} \), \( B = \{\text{her first child is affected}\} \), \( D = \{\text{at least one of her five children is affected}\} \). Note: let's assume that "carrier" means having either genotype +d or dd.

   Assume that \( p = 1\% \), and calculate the following.

   a. \( \Pr(C | U) \)
   
   b. \( \Pr(C | U \text{ and } A) \)
   
   c. \( \Pr(B | U \text{ and } A) \)
d. \( \Pr(D \mid U \text{ and } A) \)

3. Consider two urns. Urn A contains 5 green balls and 5 blue balls. Urn B contains 2 green balls and 8 blue balls. I roll a six-sided, fair die. If I get a 3 or a 4, I draw two balls without replacement from urn A; otherwise, I draw two balls without replacement from urn B.

Let \( A = \{\text{draws are made from urn A}\} \), \( B = \{\text{draws are made from urn B}\} \), \( G_1 = \{\text{first ball is green}\} \) and \( G_2 = \{\text{second ball is green}\} \).

Calculate the following.

a. \( \Pr(G_1) \)

b. \( \Pr(G_2 \mid A \text{ and } G_1) \)

c. \( \Pr(G_2 \mid A) \)

d. \( \Pr(G_2 \mid B \text{ and } G_1) \)

e. \( \Pr(G_2 \mid G_1) \)

f. \( \Pr(\text{exactly one green ball}) \)

g. \( \Pr(A \mid \text{at least one green ball}) \)

4. Suppose that a mare is corralled with two stallions: a champion and a dud. The mare gets pregnant and produces a colt.

a. What is \( \Pr(\text{the champion is the colt's father}) \)?

b. Suppose that the champion is known to carry a rare marker on its Y chromosome, present in only 2% of male horses. Suppose that the colt is found to also carry this Y marker. What is \( \Pr(\text{the champion is the colt's father} \mid \text{this information}) \)?

c. Suppose that, in addition to the conditions in (b), the mare had been "exposed" to another 998 stallions. What is \( \Pr(\text{the champion is the colt's father} \mid \text{this information}) \)?