Statistics for laboratory scientists

Solutions for the homework problems for lecture 8

1. Let \( p = \Pr(\text{male}) = 105/205 = \text{(approx) 0.512} \). Let \( X \) = number of males in 6 random newborns. The \( X \sim \text{binomial}(n=6, p=0.512) \). Let 
\[
p(x) = \Pr(X=x)
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( p(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.3%</td>
</tr>
<tr>
<td>1</td>
<td>8.5%</td>
</tr>
<tr>
<td>2</td>
<td>22.3%</td>
</tr>
<tr>
<td>3</td>
<td>31.2%</td>
</tr>
<tr>
<td>4</td>
<td>24.6%</td>
</tr>
<tr>
<td>5</td>
<td>10.3%</td>
</tr>
<tr>
<td>6</td>
<td>1.8%</td>
</tr>
</tbody>
</table>

In R, type \( \text{round(dbinom(0:6, 6, 105/205)*100, 1)} \)

2. Let \( n = \text{number of slides examined} \) and \( X = \text{number that are positive}. \)
If the sample is positive, then \( X \sim \text{binomial}(n, p=0.2) \).

We seek \( n \) such that \( \Pr(X = 0) \leq 1\% \). But \( \Pr(X=0) = (1-p)^n = (0.8)^n \)

Thus, we wish to solve the following equation for \( n \): \( (0.8)^n \leq 0.01 \).

The solution: take logs.

\[
(0.8)^n \leq 0.01 \\
\ln{(0.8)^n} \leq \ln{0.01} \\
n \ln{(0.8)} \leq \ln{0.01} \\
n (-0.223) \leq (-4.605) \\
n \geq 20.6
\]

Thus, we must examine at least 21 slides.
That is a lot of work! An improved method would be recommended.

3. 
   a. Let $S = \{\text{fly has singed bristles}\}$ and $W = \{\text{fly has white eyes}\}$. $S$ and $W$ are independent, $\Pr(S) = 1/2$, and $\Pr(W) = 1/4$.

   $\Pr(S \text{ and } W) = \Pr(S) \Pr(W) = (1/2) \times (1/4) = 1/8 = \text{(approx)} 13\%$.

   b. Let $W_i = \{\text{fly } i \text{ has white eyes}\}$. The $W_i$ are independent, and $\Pr(W_i) = 1/4$.

   $\Pr(\text{all four have white eyes}) = \Pr(W_1 \text{ and } W_2 \text{ and } W_3 \text{ and } W_4) = (1/4)^4 = \text{(approx) } 4/1000$.

   c. First, note that $\Pr(\text{a fly has neither white eyes nor singed bristles}) = \Pr(\text{not W and not S}) = \Pr(\text{not W}) \times \Pr(\text{not S}) = (3/4) \times (1/2) = 3/8$.

   $\Pr(\text{none of four flies have either white eyes or singed bristles}) = (3/8)^4 = \text{(approx) } 2\%$.

   d. $\Pr(\text{at least one of two flies has white eyes or singed bristles or both}) = 1 - \Pr(\text{neither has white eyes or singed bristles or both})$

   $= 1 - \Pr(\text{both are not W and not S}) = 1 - (3/8)^2 = \text{(approx) } 86\%$.

4. $\Pr(\text{exactly 50 heads in 100 tosses}) = \binom{100}{50} (0.5)^{100} = \text{(approx) } 8\%$.

   $\Pr(\text{exactly 3 heads in 10 tosses}) = \binom{10}{3} (0.5)^{10} = \text{(approx) } 12\%$.

   Thus, the latter is more likely.

   You can use the following R code, if your hand calculator is not sufficiently advanced: `dbinom(50,100,0.5)` and `dbinom(3,10,0.5)`

5. $\Pr(\text{getting a double-six}) = 1/36$. Thus, if we let $X = \text{number of double-sixes in 36 rolls of a pair of fair, six-sided dice}$, $X \sim \text{binomial (n=36, } p=1/36)$. 

http://www.biostat.jhsph.edu/~kbroman/teaching/labstat/third/soln08.html
a. \( X \sim \text{binomial}(n=36, p=1/36) \)

b. \( \Pr(X=2) = \binom{36}{2} \left(\frac{1}{36}\right)^2 \left(\frac{35}{36}\right)^{34} \approx 19\% \)

c. \( E(X) = 36 \times \left(\frac{1}{36}\right) = 1 \)

d. \( SD(X) = \sqrt{36 \times \left(\frac{1}{36}\right) \times \left(\frac{35}{36}\right)} \approx 0.99 \)

e. \( \Pr(X > 2) = 1 - \Pr(X=0) - \Pr(X=1) - \Pr(X=2) \approx 8\% \).

Parts (a) and (d) would be a lot easier on the computer: R\( \text{dbinom}(2, 36, 1/36) \) and \( \text{1-pbinom}(2, 36, 1/36) \).

6. \( X \) follows a Poisson(\( \lambda = 2 \)) distribution.

a. \( E(X) = 2 \).

b. \( SD(X) = \sqrt{2} \approx 1.4 \).

c. \( \Pr(X = 0) = e^{-2} \approx 14\% \).

d. \( \Pr(X = 5) = e^{-2} \frac{2^5}{5!} \approx 3.6\% \).

e. \( \Pr(X > 2) = 1 - \Pr(X = 0) - \Pr(X = 1) - \Pr(X = 2) = 1 - e^{-2} - e^{-2} \frac{2^2}{2!} \approx 32\% \).

7. \( E(Y) = 30 \) and \( SD(Y) = 5 \).

a. \( Z = (X - 30)/5. \ E(Z) = 0 \) and \( SD(Z) = 1 \).

b. \( X = -Y. \ E(X) = -E(Y) = -30 \) and \( SD(X) = SD(Y) = 5 \).

c. \( R = 5 + Y/3. \ E(R) = 5 + E(Y)/3 = 15 \) and \( SD(R) = SD(Y)/3 = \approx 1.67 \).

8. \( U \sim \text{uniform}(5, 10) \).

a. \( E(U) = (5+10)/2 = 7.5 \)

b. \( \Pr(U = 6) = 0 \) (\( U \) is a continuous random variable.)

c. \( \Pr(U > 6) = 4/5 = 80\% \)
d. \( \Pr(7 < U < 9) = \frac{2}{5} = 40\% \)