JOHNS HOPKINS BLOOMBERG SCHOOL of PUBLIC HEALTH

Statistics for laboratory scientists

Solutions for the homework problems for lecture 8

1. Let p = Pr(male) = 105/205 = (approx) 0.512. Let X = number of males in 6 random newborns. The X ~ binomial(n=6, p=0.512). Let p(x) = Pr(X=x)

x	p(x)
0	1.3%
1	8.5%
2	22.3%
3	31.2%
4	24.6%
5	10.3%
6	1.8%

   In R, type `round(dbinom(0:6, 6, 105/205)*100, 1)`

2. Let n = number of slides examined and X = number that are positive. If the sample is positive, then X ~ binomial(n, p=0.2).

   We seek n such that Pr(X = 0) <= 1%. But Pr(X=0) = $(1-p)^n$ = $(0.8)^n$

   Thus, we wish to solve the following equation for n: $(0.8)^n <= 0.01$.

   The solution: take logs.

   $$(0.8)^n <= 0.01$$
   $$\ln\{(0.8)^n\} <= \ln\{0.01\}$$
   $$n \ln\{(0.8)\} <= \ln\{0.01\}$$
   $$n (-0.223) <= (-4.605)$$
   $$n >= 20.6$$

   Thus, we must examine at least 21 slides.

That is a lot of work! An improved method would be recommended.

3.

a. Let S = {fly has singed bristles} and W = {fly has white eyes}. S and W are independent, Pr(S) = 1/2, and Pr(W) = 1/4.

   Pr(S and W) = Pr(S) Pr(W) = (1/2) × (1/4) = 1/8 = (approx) 13%.

b. Let W_i = {fly i has white eyes}. The W_i are independent, and Pr(W_i) = 1/4.

   Pr(all four have white eyes) = Pr(W_1 and W_2 and W_3 and W_4) = $(1/4)^4$ = (approx) 4/1000.

c. First, note that Pr(a fly has neither white eyes nor singed bristles) = Pr(not W and not S) = Pr(not W) × Pr(not S) = (3/4) × (1/2) = 3/8.

   Pr(none of four flies have either white eyes or singed bristles) = $(3/8)^4$ = (approx) 2%.

d. Pr(at least one of two flies has white eyes or singed bristles or both) = 1 - Pr(neither has white eyes or singed bristles or both) = 1 - Pr(both are not W and not S) = 1 - $(3/8)^2$ = (approx) 86%.

4. Pr(exactly 50 heads in 100 tosses) = (100 choose 50) $(0.5)^{100}$ = (approx) 8%.

   Pr(exactly 3 heads in 10 tosses) = (10 choose 3) $(0.5)^{10}$ = (approx) 12%.

   Thus, the latter is more likely.

   You can use the following R code, if your hand calculator is not sufficiently advanced: `dbinom(50,100,0.5)` and `dbinom(3,10,0.5)`

5. Pr(getting a double-six) = 1/36. Thus, if we let X = number of double-sixes in 36 rolls of a pair of fair, six-sided dice, $X \sim$ binomial (n=36, p=1/36).

a. X ~ binomial(n=36, p=1/36)

b. Pr(X=2) = (36 choose 2) $(1/36)^2$ $(35/36)^{34}$ = (approx) 19%

c. E(X) = 36 × (1/36) = 1

d. SD(X) = $[36 \times (1/36) \times (35/36)]^{(1/2)}$ = (approx) 0.99

e. Pr(X > 2) = 1 - Pr(X=0) - Pr(X=1) - Pr(X=2) = (approx) 8%.

Parts (a) and (d) would be a lot easier on the computer: `dbinom(2, 36, 1/36)` and `1-pbinom(2,36,1/36)`.

6. X follows a Poisson(lambda=2) distribution.

a. E(X) = 2.

b. SD(X) = sqrt(2) = (approx) 1.4.

c. Pr(X = 0) = e^{-2} = (approx) 14%.

d. Pr(X = 5) = e^{-2} 2^5 / 5! = (approx) 3.6%.

e. Pr(X > 2) = 1 - Pr(X = 0) - Pr(X = 1) - Pr(X = 2) = 1 - e^{-2} - e^{-2} 2 - e^{-2} 2^2 / 2! = (approx) 32%.

7. E(Y) = 30 and SD(Y) = 5.

a. Z = (Y - 30)/5. E(Z)=0 and SD(Z)=1.

b. X = - Y. E(X) = -E(Y) = -30 and SD(X) = SD(Y) = 5.

c. R = 5 + Y/3. E(R) = 5 + E(Y)/3 = 15 and SD(R) = SD(Y)/3 = (approx) 1.67.

8. U ~ uniform(5, 10).

a. E(U) = (5+10)/2 = 7.5

b. Pr(U = 6) = 0 (U is a continuous random variable.)

c. Pr(U > 6) = 4/5 = 80%

d. $\Pr(7 < U < 9) = 2/5 = 40\%$

[3rd term syllabus | 4rd term syllabus | R for Windows]

Last modified: Wed Feb 22 09:44:17 EST 2006