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Statistics for laboratory scientists

Solutions for the homework problems for lecture 8

1. Let $p = \Pr(\text{male}) = 105/205 = (\text{approx}) 0.512$. Let $X =$ number of males in 6 random newborns. The $X \sim \text{binomial}(n=6, p=0.512)$. Let $p(x) = \Pr(X=x)$

x	p(x)
0	1.3%
1	8.5%
2	22.3%
3	31.2%
4	24.6%
5	10.3%
6	1.8%

In R, type `round(dbinom(0:6, 6, 105/205)*100, 1)`

2. Let $n =$ number of slides examined and $X =$ number that are positive. If the sample is positive, then $X \sim \text{binomial}(n, p=0.2)$.

We seek n such that $\Pr(X = 0) \leq 1\%$. But $\Pr(X=0) = (1-p)^n = (0.8)^n$

Thus, we wish to solve the following equation for n : $(0.8)^n \leq 0.01$.

The solution: take logs.

$$\begin{aligned}
 (0.8)^n &\leq 0.01 \\
 \ln\{(0.8)^n\} &\leq \ln\{0.01\} \\
 n \ln\{0.8\} &\leq \ln\{0.01\} \\
 n (-0.223) &\leq (-4.605) \\
 n &\geq 20.6
 \end{aligned}$$

Thus, we must examine at least 21 slides.

That is a lot of work! An improved method would be recommended.

3.

- a. Let $S = \{\text{fly has singed bristles}\}$ and $W = \{\text{fly has white eyes}\}$.
 S and W are independent, $\Pr(S) = 1/2$, and $\Pr(W) = 1/4$.

$$\Pr(S \text{ and } W) = \Pr(S) \Pr(W) = (1/2) \times (1/4) = 1/8 = (\text{approx}) 13\%.$$

- b. Let $W_i = \{\text{fly } i \text{ has white eyes}\}$. The W_i are independent, and $\Pr(W_i) = 1/4$.

$$\Pr(\text{all four have white eyes}) = \Pr(W_1 \text{ and } W_2 \text{ and } W_3 \text{ and } W_4) = (1/4)^4 = (\text{approx}) 4/1000.$$

- c. First, note that $\Pr(\text{a fly has neither white eyes nor singed bristles}) = \Pr(\text{not } W \text{ and not } S) = \Pr(\text{not } W) \times \Pr(\text{not } S) = (3/4) \times (1/2) = 3/8$.

$$\Pr(\text{none of four flies have either white eyes or singed bristles}) = (3/8)^4 = (\text{approx}) 2\%.$$

- d. $\Pr(\text{at least one of two flies has white eyes or singed bristles or both}) = 1 - \Pr(\text{neither has white eyes or singed bristles or both}) = 1 - \Pr(\text{both are not } W \text{ and not } S) = 1 - (3/8)^2 = (\text{approx}) 86\%$.

4. $\Pr(\text{exactly 50 heads in 100 tosses}) = (100 \text{ choose } 50) (0.5)^{100} = (\text{approx}) 8\%$.

$$\Pr(\text{exactly 3 heads in 10 tosses}) = (10 \text{ choose } 3) (0.5)^{10} = (\text{approx}) 12\%.$$

Thus, the latter is more likely.

You can use the following R code, if your hand calculator is not sufficiently advanced: `dbinom(50,100,0.5)` and `dbinom(3,10,0.5)`

5. $\Pr(\text{getting a double-six}) = 1/36$. Thus, if we let $X = \text{number of double-sixes in 36 rolls of a pair of fair, six-sided dice}$, $X \sim \text{binomial}(n=36, p=1/36)$.

- a. $X \sim \text{binomial}(n=36, p=1/36)$
- b. $\Pr(X=2) = \binom{36}{2} (1/36)^2 (35/36)^{34} = (\text{approx}) 19\%$
- c. $E(X) = 36 \times (1/36) = 1$
- d. $SD(X) = [36 \times (1/36) \times (35/36)]^{(1/2)} = (\text{approx}) 0.99$
- e. $\Pr(X > 2) = 1 - \Pr(X=0) - \Pr(X=1) - \Pr(X=2) = (\text{approx}) 8\%$.

Parts (a) and (d) would be a lot easier on the computer: `dbinom(2, 36, 1/36)` and `1-pbinom(2,36,1/36)`.

- 6. X follows a Poisson($\lambda=2$) distribution.
 - a. $E(X) = 2$.
 - b. $SD(X) = \text{sqrt}(2) = (\text{approx}) 1.4$.
 - c. $\Pr(X = 0) = e^{-2} = (\text{approx}) 14\%$.
 - d. $\Pr(X = 5) = e^{-2} 2^5 / 5! = (\text{approx}) 3.6\%$.
 - e. $\Pr(X > 2) = 1 - \Pr(X = 0) - \Pr(X = 1) - \Pr(X = 2) = 1 - e^{-2} - e^{-2} 2 - e^{-2} 2^2 / 2! = (\text{approx}) 32\%$.
- 7. $E(Y) = 30$ and $SD(Y) = 5$.
 - a. $Z = (Y - 30)/5$. $E(Z)=0$ and $SD(Z)=1$.
 - b. $X = -Y$. $E(X) = -E(Y) = -30$ and $SD(X) = SD(Y) = 5$.
 - c. $R = 5 + Y/3$. $E(R) = 5 + E(Y)/3 = 15$ and $SD(R) = SD(Y)/3 = (\text{approx}) 1.67$.
- 8. $U \sim \text{uniform}(5, 10)$.
 - a. $E(U) = (5+10)/2 = 7.5$
 - b. $\Pr(U = 6) = 0$ (U is a continuous random variable.)
 - c. $\Pr(U > 6) = 4/5 = 80\%$

$$d. \Pr(7 < U < 9) = 2/5 = 40\%$$

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