Statistics for laboratory scientists

Solutions for the homework problems for lecture 9

1. X ~ normal(mean=5, SD=3). Let $Z = (X - 5)/3$.

    a. $\Pr(X < 6) = \Pr[Z < (6 - 5)/3] = \Pr(Z < 1/3)$ = (approx) **63%**.

    b. $\Pr(X > 0) = \Pr(Z > -5/3) = \Pr(Z < 5/3)$ = (approx) **95%**.

    c. $\Pr(0 < X < 5) = \Pr(X > 0) - 1/2$ = (approx) **45%**.

    d. $\Pr(2 < X < 8) = \Pr[(2-5)/3 < Z < (8-5)/3] = \Pr(-1 < Z < 1)$ = (approx) **68%**.

    e. $\Pr(|X - 5| > 2) = \Pr(|Z| > 2/3) = 2 \times \Pr(Z < -2/3)$ = (approx) **50%**.

2. Y ~ normal(mean=200, SD=18). Let $Z = (Y - 200)/18$.

    a. $\Pr(Y > 250) = \Pr[Z > (250-200)/18] = 1 - \Pr[Z < (250-200)/18]$ = (approx) **3/1000**.

    b. $\Pr(180 < Y < 220) = \Pr(-20/18 < Z < 20/18) = \Pr(Z < 20/18) - \Pr(Z < -20/18)$ = (approx) **73%**.

    c. $\Pr(|Y - 180| > 20) = \Pr(Y > 200) + \Pr(Y < 160) = 50\% + \Pr[Z < (160 - 200)/18]$ = (approx) **51%**.

3. Let L ~ normal(mean=3.2, SD=0.8). Let $Z = (L - 3.2)/0.8$.

    a. $\Pr(L > 4.5) = \Pr[Z > (4.5 - 3.2)/0.8] = 1 - \Pr(Z < 1.625)$ = (approx) **5%**.

    b. $\Pr(L > 1.78) = 1 - \Pr[Z < (1.78 - 3.2)/0.8]$ = (approx) **96%**.

    c. $\Pr(2.9 < L < 3.6) = \Pr[(2.9-3.2)/0.8 < Z < (3.6-3.2)/0.8]$ = (approx) **34%**.

4. X and Y are independent, X ~ binomial(n=5, p=0.1), and Y ~ binomial(n=5, p=0.4).

Thus, E(X) = 0.5 and SD(X) = sqrt{5 × 0.1 × 0.9} = (approx) 0.67.

Similarly, E(Y) = 2.0 and SD(Y) = (approx) 1.10.

a. E(X + Y) = E(X) + E(Y) = 0.5 + 2.0 = **2.5**.

b. Since X and Y are independent, SD(X + Y) = sqrt{SD(X)^2 + SD(Y)^2 = sqrt{0.67^2 + 1.10^2} = (approx) **1.28**.

c. E[(X+Y)/2] = 2.5/2 = **1.25**

d. SD[(X+Y)/2] = (approx) 1.28/2 = **0.64**.

e. Note that X - Y = X + (-Y), and that E(-Y) = -E(Y) = -2.5 and SD(-Y) = E(Y) = (approx) 1.10.

   Thus, E(X-Y) = 0.5 - 2.0 = **-1.5**.

f. SD(X - Y) = SD(X + Y) = (approx) **1.28**.

5. X_1, X_2, X_3, ..., X_{10} are iid with mean=3 and SD=3.

   a. E(X_1 + X_2 + ... + X_{10}) = 10 × 3 = **30**.

   b. SD(X_1 + X_2 + ... + X_{10}) = sqrt{10} × 3 = (approx) **9.5**.

   c. E[(X_1 + X_2 + ... + X_{10})/10] = **3**.

   d. SD[(X_1 + X_2 + ... + X_{10})/10] = 3 / sqrt{10} = (approx) **0.95**.

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