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## Statistics for laboratory scientists

### Solutions for the homework problems for lecture 9

1.  $X \sim \text{normal}(\text{mean}=5, \text{SD}=3)$ . Let  $Z = (X - 5)/3$ .
  - a.  $\Pr(X < 6) = \Pr[Z < (6 - 5)/3] = \Pr(Z < 1/3) = (\text{approx})$  **63%**.
  - b.  $\Pr(X > 0) = \Pr(Z > -5/3) = \Pr(Z < 5/3) = (\text{approx})$  **95%**.
  - c.  $\Pr(0 < X < 5) = \Pr(X > 0) - 1/2 = (\text{approx})$  **45%**.
  - d.  $\Pr(2 < X < 8) = \Pr[(2-5)/3 < Z < (8-5)/3] = \Pr(-1 < Z < 1) = (\text{approx})$  **68%**.
  - e.  $\Pr(|X - 5| > 2) = \Pr(|Z| > 2/3) = 2 \times \Pr(Z < -2/3) = (\text{approx})$  **50%**.
  
2.  $Y \sim \text{normal}(\text{mean}=200, \text{SD}=18)$ . Let  $Z = (Y - 200)/18$ .
  - a.  $\Pr(Y > 250) = \Pr[Z > (250-200)/18] = 1 - \Pr[Z < (250-200)/18] = (\text{approx})$  **3/1000**.
  - b.  $\Pr(180 < Y < 220) = \Pr(-20/18 < Z < 20/18) = \Pr(Z < 20/18) - \Pr(Z < -20/18) = (\text{approx})$  **73%**.
  - c.  $\Pr(|Y - 180| > 20) = \Pr(Y > 200) + \Pr(Y < 160) = 50\% + \Pr[Z < (160 - 200)/18] = (\text{approx})$  **51%**.
  
3. Let  $L \sim \text{normal}(\text{mean}=3.2, \text{SD}=0.8)$ . Let  $Z = (L - 3.2)/0.8$ .
  - a.  $\Pr(L > 4.5) = \Pr[Z > (4.5 - 3.2)/0.8] = 1 - \Pr(Z < 1.625) = (\text{approx})$  **5%**.
  - b.  $\Pr(L > 1.78) = 1 - \Pr[Z < (1.78 - 3.2)/0.8] = (\text{approx})$  **96%**.
  - c.  $\Pr(2.9 < L < 3.6) = \Pr[(2.9-3.2)/0.8 < Z < (3.6-3.2)/0.8] = (\text{approx})$  **34%**.
  
4.  $X$  and  $Y$  are independent,  $X \sim \text{binomial}(n=5, p=0.1)$ , and  $Y \sim \text{binomial}(n=5, p=0.4)$ .

Thus,  $E(X) = 0.5$  and  $SD(X) = \sqrt{5 \times 0.1 \times 0.9} = (\text{approx}) 0.67$ .

Similarly,  $E(Y) = 2.0$  and  $SD(Y) = (\text{approx}) 1.10$ .

a.  $E(X + Y) = E(X) + E(Y) = 0.5 + 2.0 = \mathbf{2.5}$ .

b. Since  $X$  and  $Y$  are independent,  $SD(X + Y) = \sqrt{SD(X)^2 + SD(Y)^2} = \sqrt{0.67^2 + 1.10^2} = (\text{approx}) \mathbf{1.28}$ .

c.  $E[(X+Y)/2] = 2.5/2 = \mathbf{1.25}$

d.  $SD[(X+Y)/2] = (\text{approx}) 1.28/2 = \mathbf{0.64}$ .

e. Note that  $X - Y = X + (-Y)$ , and that  $E(-Y) = -E(Y) = -2.5$  and  $SD(-Y) = E(Y) = (\text{approx}) 1.10$ .

Thus,  $E(X - Y) = 0.5 - 2.0 = \mathbf{-1.5}$ .

f.  $SD(X - Y) = SD(X + Y) = (\text{approx}) \mathbf{1.28}$ .

5.  $X_1, X_2, X_3, \dots, X_{10}$  are iid with mean=3 and SD=3.

a.  $E(X_1 + X_2 + \dots + X_{10}) = 10 \times 3 = \mathbf{30}$ .

b.  $SD(X_1 + X_2 + \dots + X_{10}) = \sqrt{10} \times 3 = (\text{approx}) \mathbf{9.5}$ .

c.  $E[(X_1 + X_2 + \dots + X_{10})/10] = \mathbf{3}$ .

d.  $SD[(X_1 + X_2 + \dots + X_{10})/10] = 3 / \sqrt{10} = (\text{approx}) \mathbf{0.95}$ .