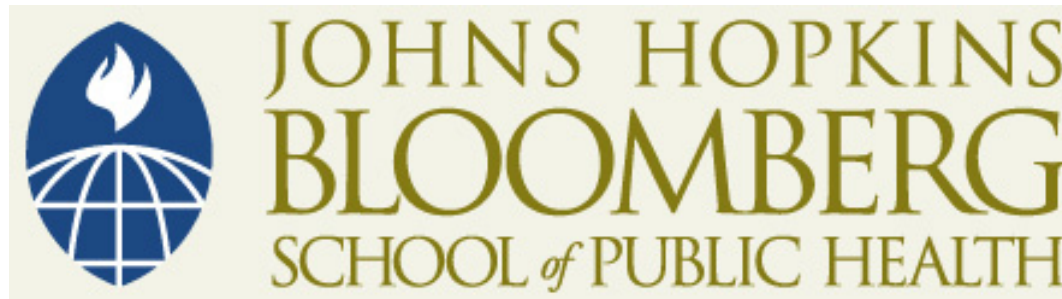


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Example

Growth of Hudson River striped bass; years 1965–1976.

y_t = incremental growth rate in year t

x_{1t} = population density in year t

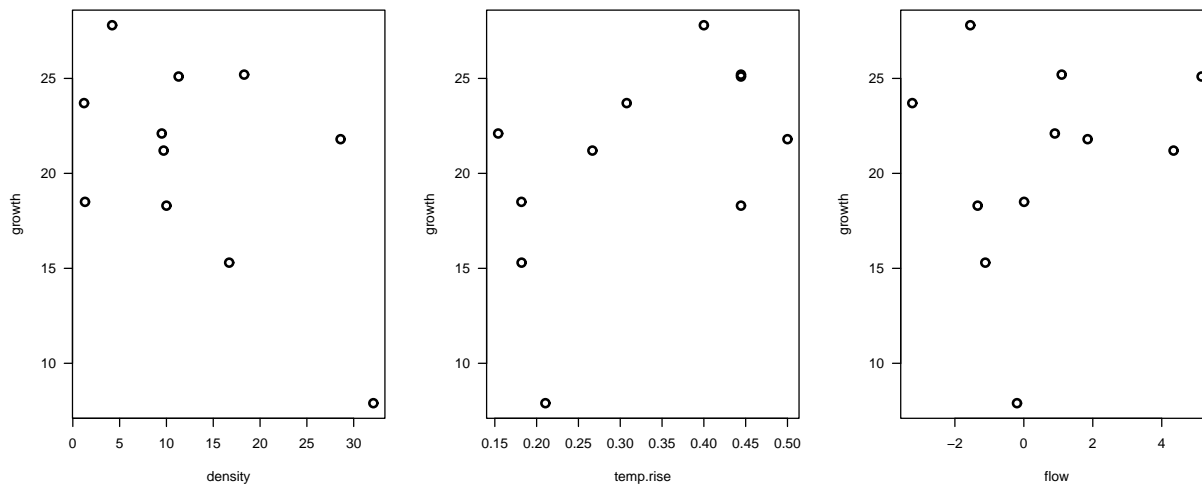
x_{2t} = rise in temperature in year t

x_{3t} = a measure of river flow in year t

Fit the model

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \epsilon_t \quad \text{where } \epsilon_t \sim \text{iid Normal}(0, \sigma^2)$$

Plot y vs each x



Results

	Est	SE	t-value	P value
Intercept	17.00	3.22	5.3	0.001
density	-0.37	0.11	-3.2	0.015
temp.rise	25.36	9.01	2.8	0.026
flow	0.45	0.47	1.0	0.362

Diagnostics

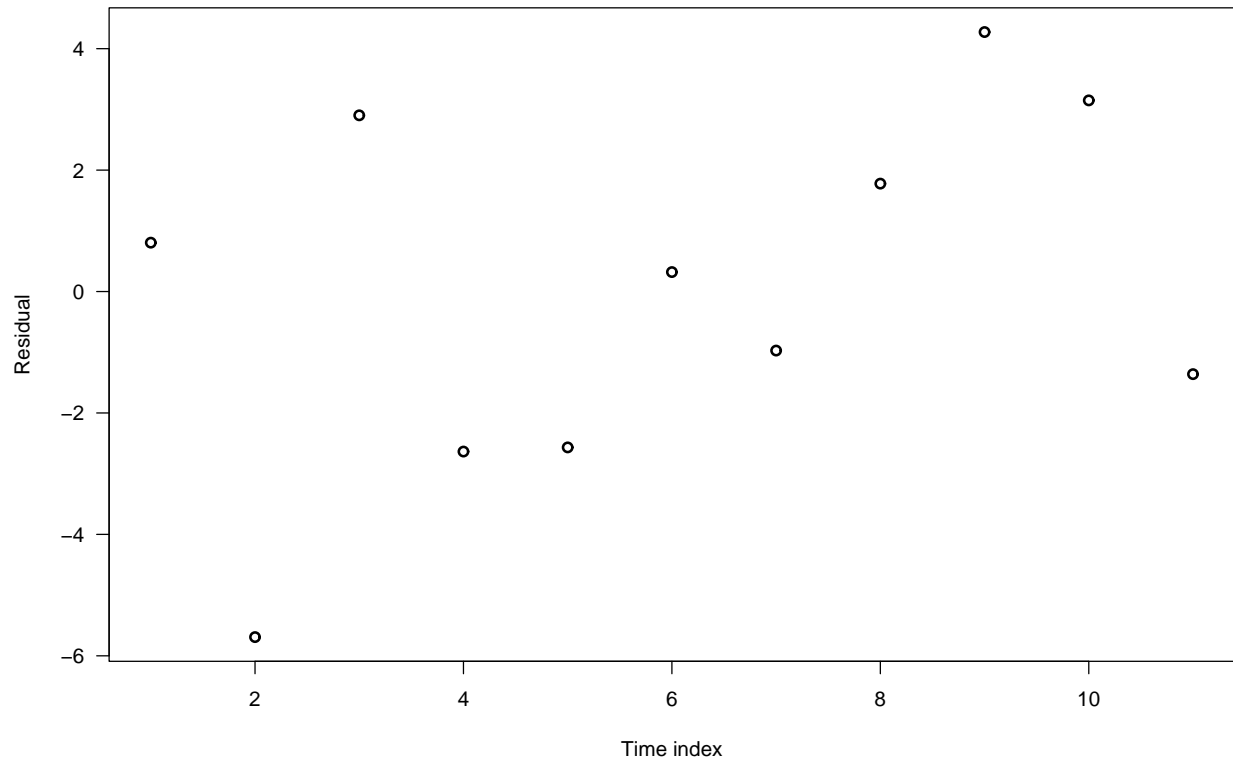
Assumptions

ϵ 's independent
 ϵ 's normally distributed
 ϵ 's have constant SD
 y 's linear in each of the x 's
No other x 's belong in the model

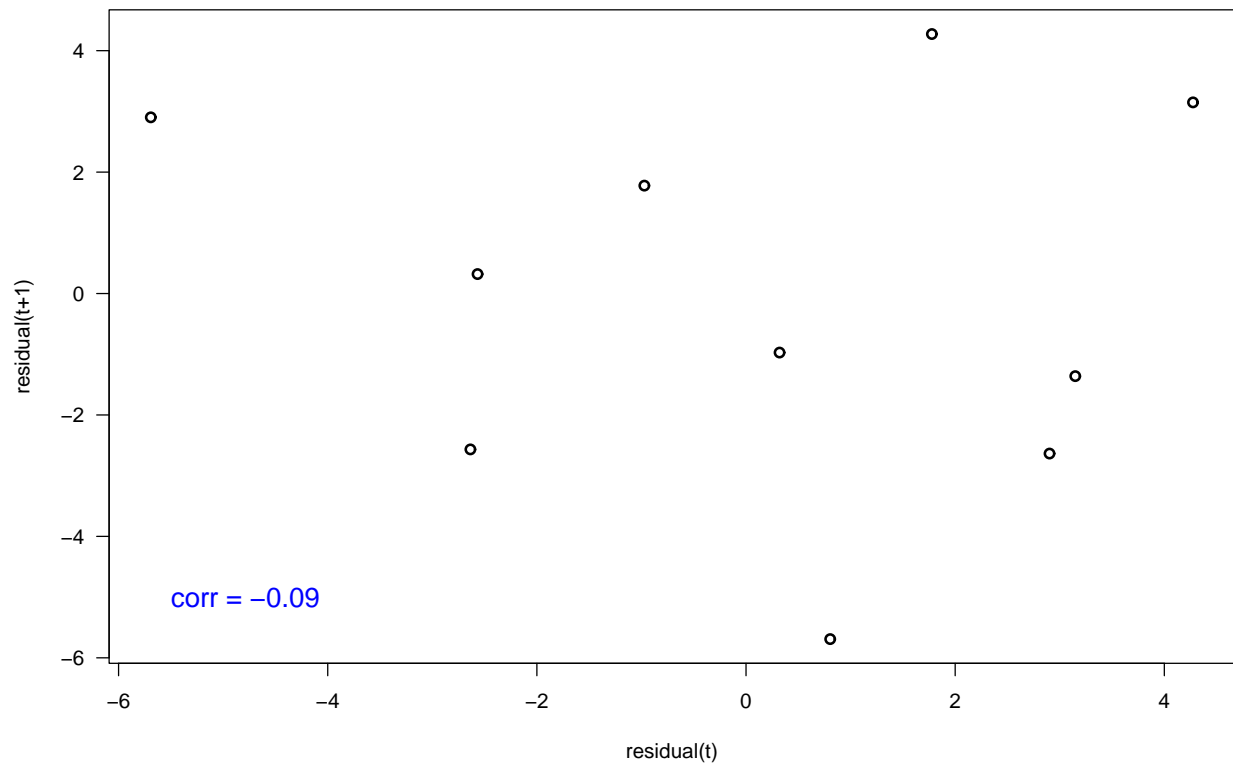
Diagnostics

Plot residuals vs time
QQ plot of residuals
Plot residuals vs fitted values
Plot residuals vs each x
Plot residuals vs other x 's

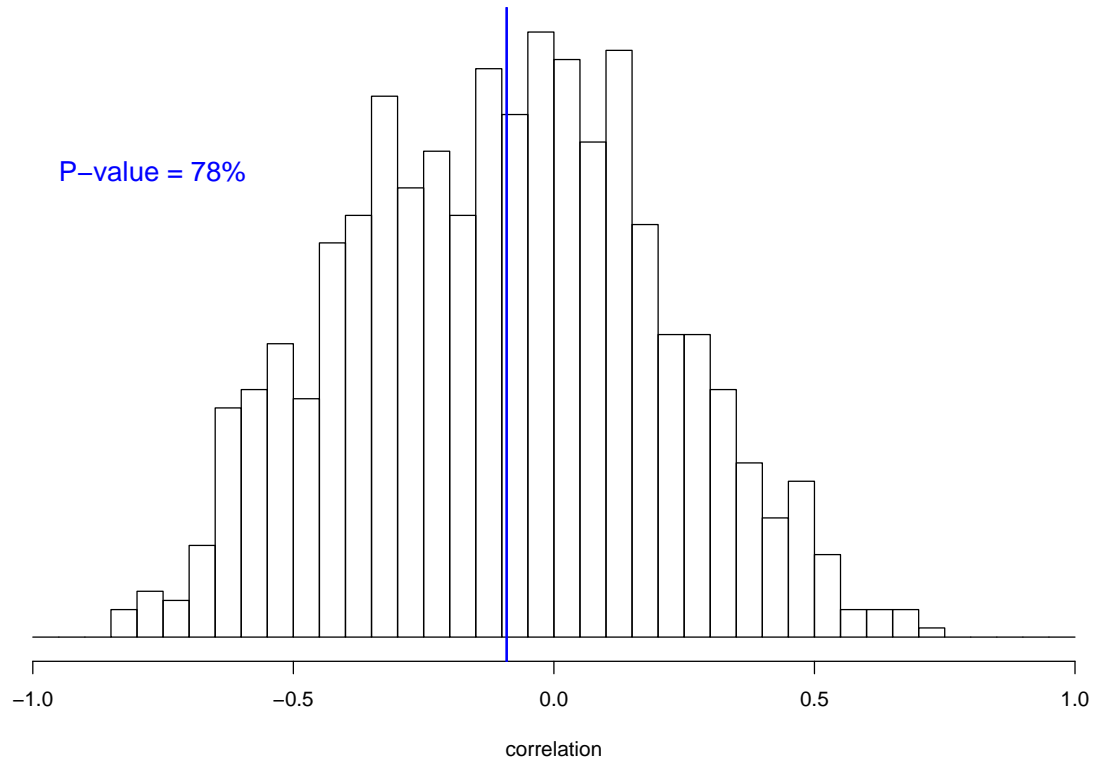
residuals vs time



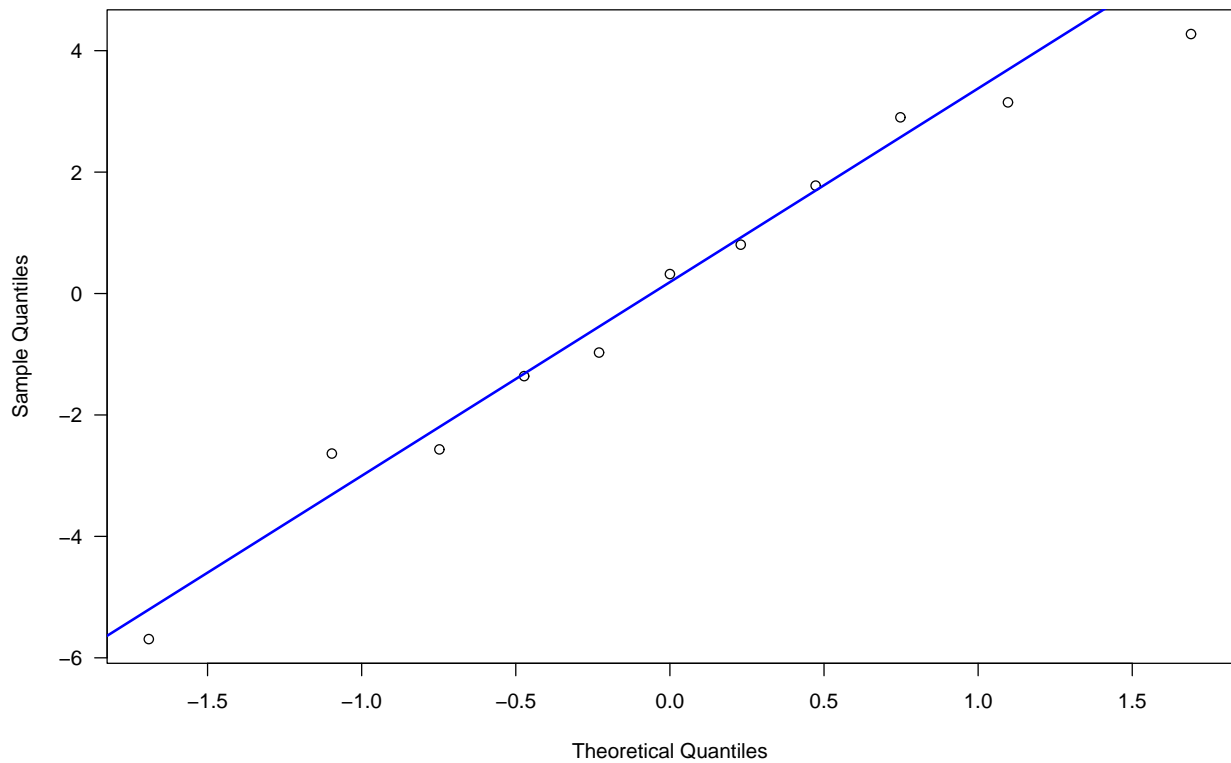
residual(t) vs residual(t+1)



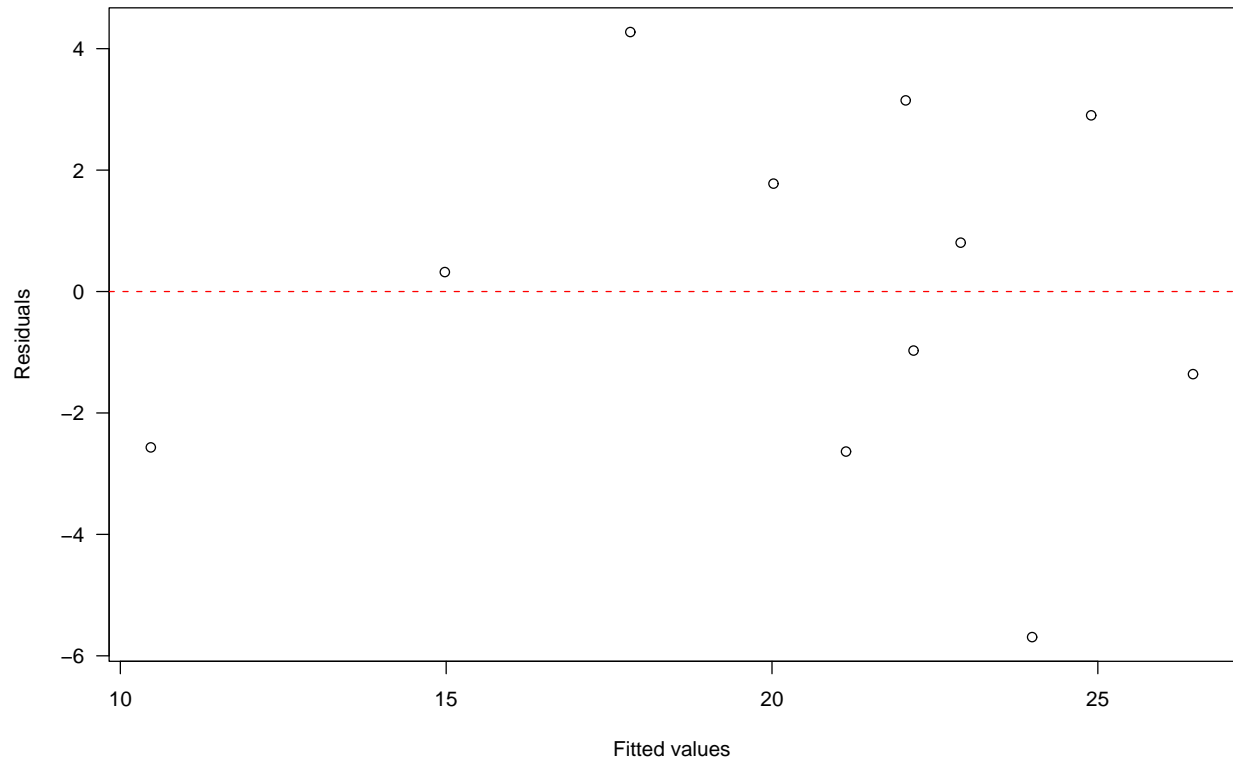
Permutation test



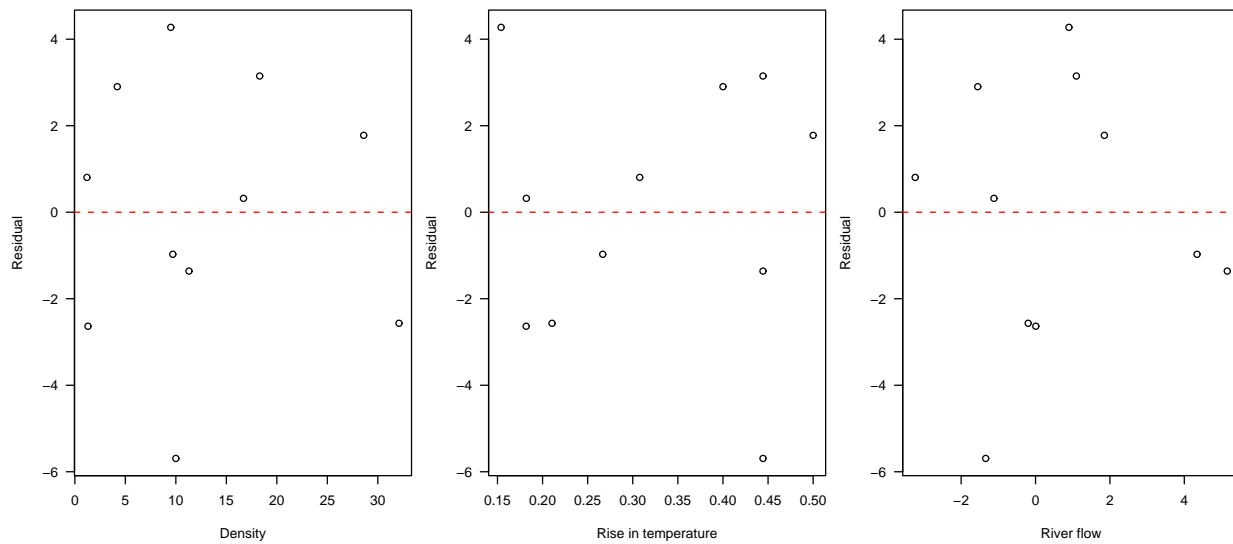
QQ plot of residuals



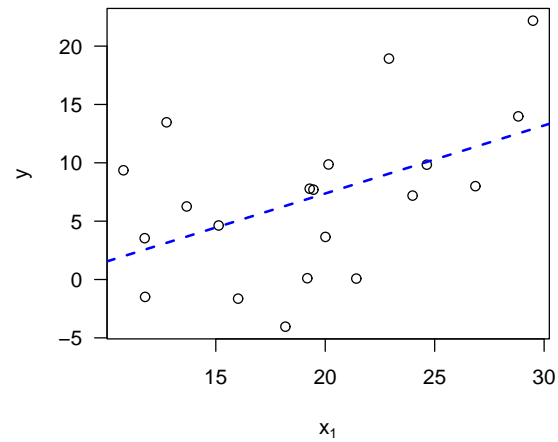
Residuals vs fitted values



Residuals vs x's

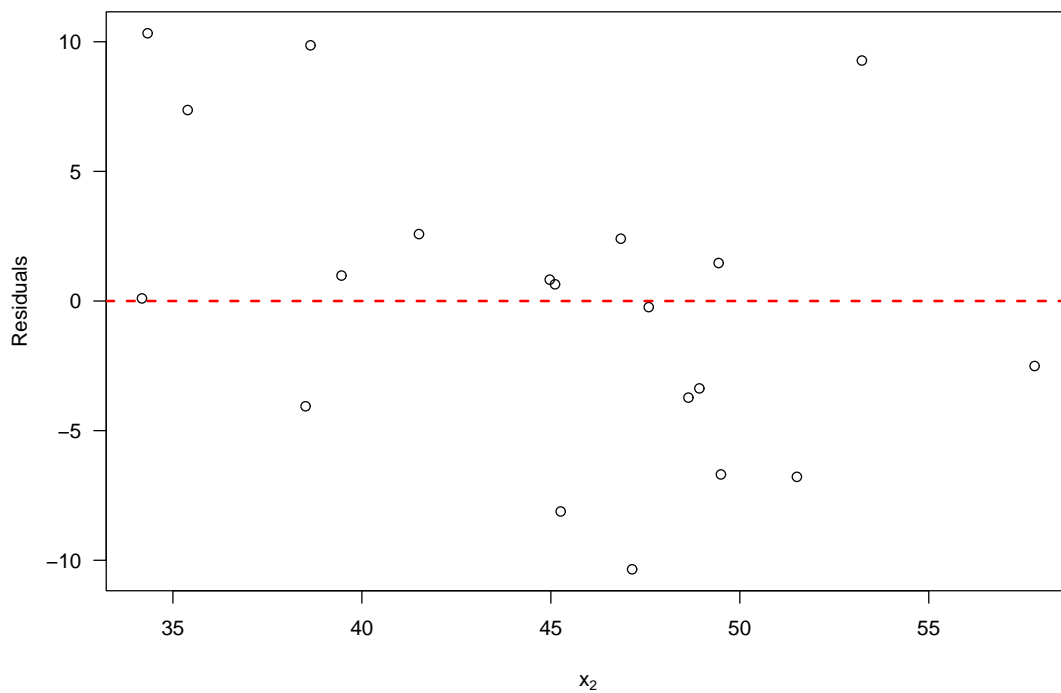


A fake example



	Est	SE	t-val	P-val
Intercept	-4.29	4.97	-0.86	0.40
x_1	0.58	0.25	2.36	0.03

Residuals vs another x



Regress y on both x's

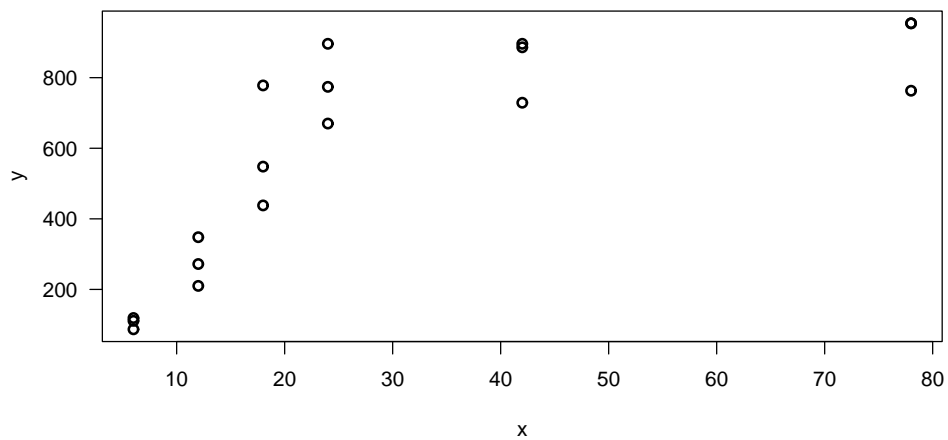
	Est	SE	t-val	P-val
Intercept	18.02	8.50	2.12	0.0491
x1	1.21	0.29	4.13	0.0007
x2	-0.77	0.26	-3.00	0.0080

One last example

Sediment ingestion by the mud snail, *Hyrobia minuta*.

y = Amount ingested

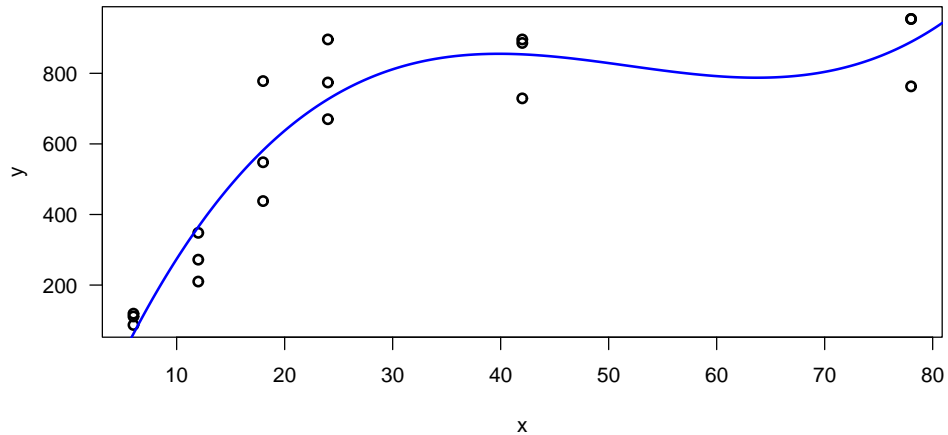
x = Time allowed to eat



A model

Let's consider the model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \epsilon_i \quad \text{where } \epsilon_t \sim \text{iid Normal}(0, \sigma^2)$$



Estimated coefficients

	Est	SE	t-val	P-val
Intercept	-339	127	-2.66	0.019
time	75.7	15.4	4.91	<0.001
time ²	-1.55	0.48	-3.22	0.006
time ³	0.010	0.004	2.52	0.024

Diagnostic plots

