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Population Change and Projection

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Section A

Rate of Change, Doubling Time, and the Relationship between Age Distinction and Demographic Rates

Population Change

- Book-keeping equation
- Let P_t = Population at time "t"
  P_0 = Population at an earlier time "0"
  B = Births between time "0" and time "t"
  D = Deaths between time "0" and "t"
  I = In-migration / immigration between time "0" and "t"
  O = Out-migration / emigration between time "0" and "t"

Continued

Population Change

- The population changes by adding births and in-migrants and subtracting deaths and out-migrants

$$P_t = P_0 + B - D + I - O$$

- Let b = Number added per time unit

$$b = \frac{P_t - P_0}{t}$$

Linear Change

- The population size changes by exactly the same amount "b" during each time period "t"

$$P_t = P_0 + bt$$

Linear Annual Rate of Change

Beginning of Period Approximation

- r_i = linear annual rate of change (beginning of period approximation)

$$r_i = \frac{P_t - P_0}{tP_0}$$

$$P_t = P_0 + r_i * t * P_0$$

Linear Annual Rate of Change

Arithmetic or Mid-Point Approximation

- r_m = linear annual rate of change (arithmetic or mid-point approximation)

$$r_m = \frac{b}{1/2(P_o + P_t)} = \frac{P_t - P_0}{t/2(P_o + P_t)}$$

Exercise

Linear Annual Growth Rate

- The Mexican population was estimated at 77,938,288 at the 1985 census and at 91,158,290 at the 1995 census
- Calculate the linear annual growth rate using both types of approximations

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, *1992 and 1997*

Exercise Answer

Linear Annual Growth Rate

- The correct answers are as follows:

r_i	r_m
1.70 %	**1.56 %**

Problem with the Linear-Change Approach

- Adding a fixed amount in each time period $\Rightarrow$ the rate of change decreases because b / Pt -> 0
- Say b $= P_0$
- Then $P_1 = P_0 + P_0 = 2P_0$

  $P_2 = P_1 + P_0 = 3P_0$

Continued

Problem with the Linear-Change Approach

- So

$$r_1 = \frac{P_1 - P_0}{1 * P_0} = 1$$

$$r_{m_1} = \frac{P_1 - P_0}{1/2(P_0 + P_1)}$$

$$= \frac{P_0}{3/2\, P_0} = \frac{2}{3}$$

Continued

Problem with the Linear-Change Approach

- And each continues declining

$$r_2 = \frac{P_2 - P_1}{1 * P_1} = \frac{3P_0 - 2P_0}{2P_0} = \frac{1}{2}$$

$$r_{m_2} = \frac{P_2 - P_1}{1/2(P_1 + P_2)} = \frac{P_0}{5/2P_0} = \frac{2}{5}$$

Geometric Change

- The population changes by step, i.e., the increment added (or the decrement is subtracted) periodically

$$P_t = P_0(1 + r_g)^t$$

- Where r_g = geometric rate of growth

Geometric Annual Rate of Change

$$r_g = \sqrt[t]{\frac{P_t}{P_0}} - 1 = e^{\frac{1}{t}\ln\left(\frac{P_t}{P_0}\right)} - 1$$

Exercise

Geometric Annual Rate of Change

- The Mexican population was estimated at 77,938,288 at the 1985 census and at 91,158,290 at the 1995 census
- Calculate the geometric annual growth rate

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, *1992 and 1997*

Exercise Answer

Geometric Annual Rate of Change

- The correct answer for r_g is as follows:
  - **1.58%**

Exponential Change

- The population changes instantaneously and continuously

$$P_t = P_0 e^{rt}$$

Exponential Growth Rate

- Exponential growth rate

$$r = \frac{1}{t} \ln\left(\frac{P_t}{P_0}\right)$$

- The advantage of the exponential method is that it's easy to solve for any parameter

$$t = \frac{\ln\left(\frac{P_t}{P_0}\right)}{r}$$

$$P_0 = \frac{P_t}{e^{rt}}$$

Exercise

Exponential Growth Rate

- The Mexican population was estimated at 77,938,288 at the 1985 census and at 91,158,290 at the 1995 census
- Calculate the exponential growth rate

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, *1992 and 1997*

Exercise Answer

Exponential Growth Rate

- The correct answer for “r” is as follows:
  - **1.57%**

Relation Between Rates of Change

Geometric and Exponential

$$r_g = e^r - 1$$

$$\text{since} \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$\text{then} \quad r_g = r + \frac{r^2}{2!} + \ldots$$

$$\Rightarrow \quad r_g > r$$

Rate of Natural Increase

- Let B = Births during a calendar year

  D = Deaths during same year

  P = Midyear population

  b = Birth rate

  d = Death rate

Continued

Rate of Natural Increase

- Most direct indication of how rapidly a population actually grew as the result of vital processes

$$r = \frac{B - D}{P} * 1000$$
$$= b - d$$

$$r < 0 \Rightarrow D > B$$
$$r = 0 \Rightarrow B = D$$
$$r > 0 \Rightarrow B > D$$

- Is influenced by age structure

Doubling Time

- Calculation based on exponential growth ("r" fixed in time)
- Allows determination of how long it will take a population to double in size (i.e., time for P_t / P_0 = 2)

Continued

Doubling Time

- Since

$$t = \frac{\ln\left(\frac{P_t}{P_0}\right)}{r}$$

- Then for doubling time

$$t = \frac{\ln(2)}{r}$$

Continued

Doubling Time

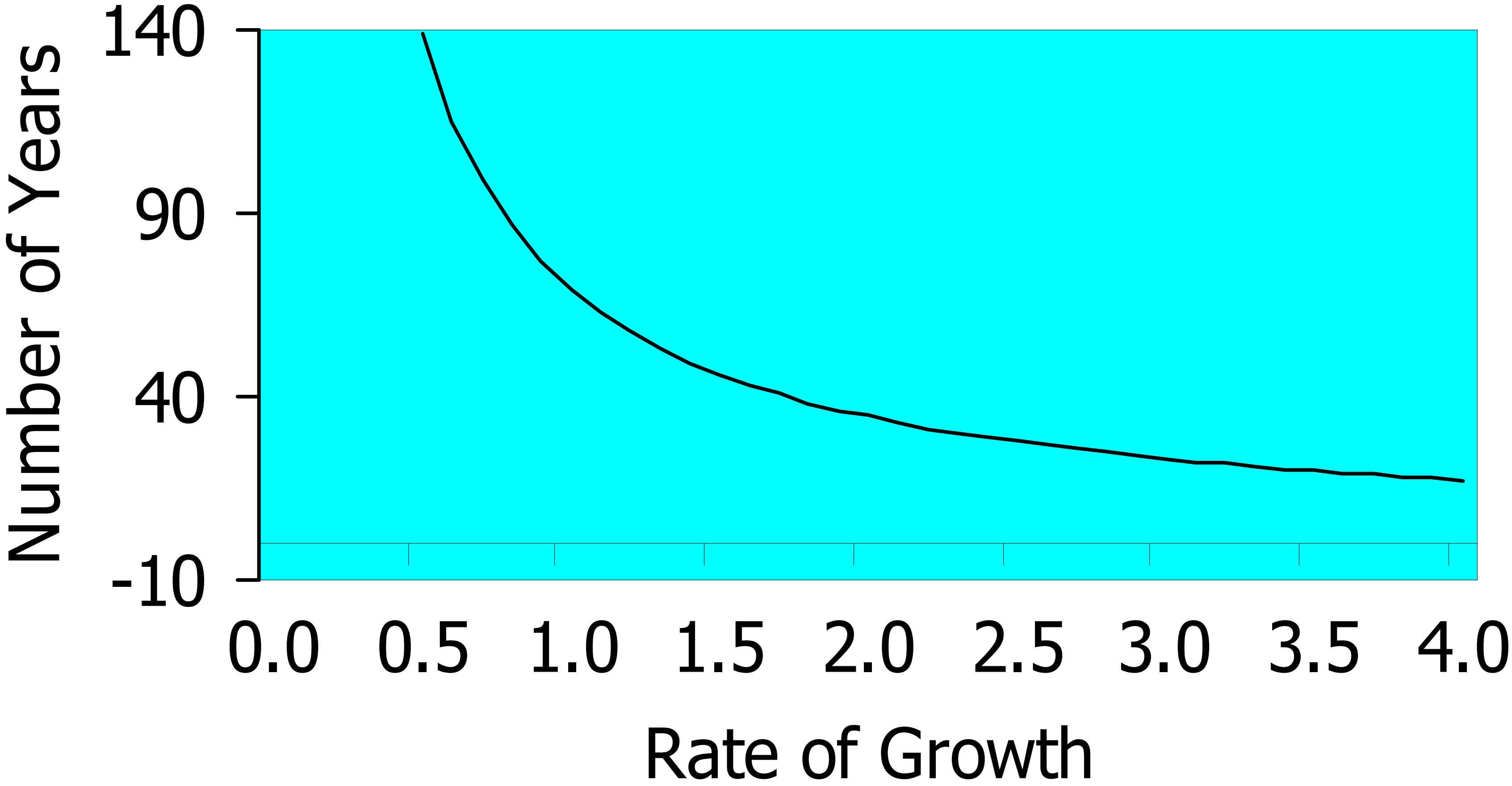
- Approximation

$$t = \frac{70}{r * 100}$$

Some Examples of Doubling Times

Rate of Growth	Number of Years	Rate of Growth	Number of Years
1.0	69.3	2.0	34.7
1.1	63.0	2.1	33.0
1.2	57.8	2.5	27.7
1.3	53.3	3.0	23.1
1.4	49.5	3.5	19.8
1.5	46.2	4.0	17.3

Number of Years Necessary for the Population to Double at Given Fixed Annual Rates of Growth



Exercise

Doubling and Tripling Time

- The Mexican population was estimated at 77,938,288 at the 1985 census and at 91,158,290 at the 1995 census
- Assuming Mexico's population is growing exponentially, what is its doubling time?
- In what year will the mid-1975 population have tripled?

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, *1992 and 1997*

Exercise Answer

Doubling and Tripling Time

- The correct answers are as follows:
  - **Doubling time = 44.2 years**
  - **Tripling time = 70.1 years**
  - **Year will triple = 2045**

The Relationship between Age Distribution and Demographic Rates

- Age distribution of a population gives a record of the demographic history of that population

Continued

The Relationship between Age Distribution and Demographic Rates

- Age distribution is sensitive to changes in demographic rates, particularly fertility and, for small areas, migration rates
- In a closed population (i.e., no migration) the age distribution is determined only by fertility and mortality

Summary

- Population change is measured by the difference between population sizes at different dates
- As time goes by, the population in an area can do the following:
  - Grow by adding births or people moving in the area (in-migrants)
  - Or it can decrease because of deaths or people moving out of the area (out-migrants)

Continued

Summary

- The simplest method to estimate the population at a later time from that of an earlier date is the bookkeeping method, which adds the births and in-migrants to the initial population and subtracts the deaths and out-migrants

Continued

Summary

- Mathematical models have been developed to estimate the average annual rate of change based on different assumptions on the way the population has grown during the period

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Section B

Projection

Population Projection

- *Population Projection*—Forecast of population change using estimates of fertility, mortality, and migration
- Projections may extend for varying numbers of years into the future
- Note:
  - Extrapolation = projection
  - Interpolation = estimation

Continued

Population Projection

- Long-term projections (over 25 years) are used in connection with the development of natural resources, planning for provision of food, for transportation and recreational facilities, etc.
- Middle-range projections (10–25 years) are used for planning educational and medical facilities and services, housing needs, etc.

Component Method

- Has become the standard methodology for projection
- Makes explicit the assumptions regarding the components of population growth—mortality, fertility, and net migration
- Gives insight into the way population changes

Continued

Component Method

- Allows the user to estimate the effect of alternative levels of fertility, mortality, or migration on population growth
- Used to obtain projections of age-sex structure

General Principles

- Start with the population distributed by age and sex at base date
- Apply assumed survival rates and age-sex specific fertility rates to obtain number of persons alive at the end of a unit of time

Continued

General Principles

- Make allowance for net migration by age and sex, if desired
- Generally five-year age groups are used
- Projection interval must be integer multiple of age interval

Population Projection

- Projection assuming closed population, constant fertility, and constant mortality
  - Let x = Age 5, 10, …, w

    t = Time "t"

    $_5L_x$ = Life table number of persons between ages "x" and "x+5" at time "t"

    $\dfrac{{}_5L_x}{{}_5L_{x-5}}$ = Survival ratio from age group (x-5, x) to age group (x, x+5)

Continued

Population Projection

$$P_{x,x+5}^{t+5} = P_{x-5,x}^{t} * \frac{{}_5L_x}{{}_5L_{x-5}}$$

- Projection of the population aged 55 to 59 from 1990 to 1995

$$P_{60-64}^{1995} = P_{55-59}^{1990} * \frac{{}_5L_{60}}{{}_5L_{55}}$$

Treatment of Open-Ended Interval

$$P_{w+}^{t+5} = P_{w-5}^{t} * \frac{{}_5L_w}{{}_5L_{w-5}} + P_{w+}^{t} * \frac{T_{(w+5)}}{T_w}$$

- Where w+ refers to the oldest age group

Continued

Treatment of Open-Ended Interval

- Keyfitz's method

$$P_{85+}^{t+5} = {}_5P_{80}^{t} * \frac{{}_5L_{85}}{{}_5L_{80}} + P_{85+}^{t} * \frac{T_{90}}{T_{85}}$$

  - In stationary population

$$\frac{{}_5P_{80}}{P_{85+}} = \frac{{}_5L_{80}}{T_{85}} \Rightarrow P_{85+} = \frac{{}_5P_{80} * T_{85}}{{}_5L_{80}}$$

Continued

Treatment of Open-Ended Interval

– Substituting

$$P_{85+}^{t+5} = {}_5P_{80}^{t} * \left(\frac{{}_5L_{85}}{{}_5L_{80}} + \frac{T_{85}}{{}_5L_{80}} * \frac{T_{90}}{T_{85}} \right)$$

$$= {}_5P_{80} * \left(\frac{{}_5L_{85} + T_{90}}{{}_5L_{80}} \right)$$

$$= {}_5P_{80} * \frac{T_{85}}{{}_5L_{80}}$$

Getting P_{0-4} at Time t+5

- Let ${}_5L_x$ = Number of survivors between ages "x" and "x+5"

  ${}_5F^f_x$ = Female fertility rate for age group (x, x+5)

  x = Age "x"

Continued

Getting $P_{0\text{-}4}$ at Time t+5

- Also let α = Beginning age of reproduction

  β = End of reproductive age

  $_5F_x$ = Female fertility rate for age group x, x+5

Continued

Getting $P_{0\text{-}4}$ at Time t+5

$_5P_0^{t+5}$ = Survivors to time "t+5" of births that occurred between "t" and "t+5"

$$= \frac{_5L_0}{2\ell_0} * \left[\sum_{x=\alpha-5}^{\beta-5} \left({}_5F_x^f + \frac{_5L_{x+5}}{_5L_x} * {}_5F_{x+5}^f \right) * {}_5P_x \right]$$

Getting $P_{0\text{-}4}$ at Time t+5 - Derivation

- Births in one year =

$$(1/2)*\left({}_5P_{15}+\frac{{}_5L_{15}}{{}_5L_{10}}*{}_5P_{10}\right)*{}_5F_{15}+$$

$$\ldots+(1/2)*\left({}_5P_{40}+\frac{{}_5L_{40}}{{}_5L_{35}}*{}_5P_{35}\right)*{}_5F_{40}$$

$$=(1/2)*\sum_{x=\alpha-5}^{\beta-5}\left({}_5F_x+\frac{{}_5L_{x+5}}{{}_5L_x}*{}_5F_{x+5}\right)*{}_5P_x$$

Continued

Getting P_{0-4} at Time t+5 - Derivation

- Births in five years =

$$5 * \left[(1/2) * \sum_{x=\alpha-5}^{\beta-5} \left({}_5F_x + \frac{{}_5L_{x+5}}{{}_5L_x} * {}_5F_{x+5} \right) * {}_5P_x \right]$$

Population Projection

- To get $P_{0\text{-}4}$, births must survive an average of 2.5 years

$$ {}_5P_0^{t+5} = \frac{5}{2} * \frac{{}_5L_0}{5\ell_0} * \left[\sum_{x=\alpha-5}^{\beta-5} \left({}_5F_x + \frac{{}_5L_{x+5}}{{}_5L_x} * {}_5F_{x+5} \right) * {}_5P_x \right] $$

$$ = \frac{{}_5L_0}{2\ell_0} * \left[\sum_{x=\alpha-5}^{\beta-5} \left({}_5F_x + \frac{{}_5L_{x+5}}{{}_5L_x} * {}_5F_{x+5} \right) * {}_5P_x \right] $$

$$ = \frac{{}_5L_0}{2\ell_0} * \left[\sum_{x=\alpha-5}^{\beta-5} \left({}_5P_x + \frac{{}_5L_{x+5}}{{}_5L_x} * {}_5P_{x+5} \right) * {}_5F_x \right] $$

Matrix Representation

- 1st row represents fertility rates and 0–5 survival
- Sub-diagonal represents survival ratios from one age group to another
- Note: The rest of the matrix (L) has zeros

Continued

Matrix Representation

$$L=\begin{bmatrix} 0 & 0 & \frac{L_0}{2\ell_0}\left(\frac{L_{15}}{L_{10}}*F_{15}\right) & \frac{L_0}{2\ell_0}\left(F_{15}+\frac{L_{20}}{L_{15}}*F_{20}\right) & \ldots & 0 & 0 \\ \frac{L_5}{L_0} & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & \frac{L_{10}}{L_5} & 0 & 0 & \ldots & 0 & 0 \\ . & . & & . & & . & . \\ . & . & & . & & . & . \\ . & . & & . & & . & . \\ 0 & 0 & 0 & 0 & \ldots & \frac{L_{w-5}}{L_{w-10}} & \frac{T_w}{T_{w-5}} \end{bmatrix}$$

Population Projection

- To get the population at a later date

$$L * P^{t} = P^{t+5}$$

$$L * P^{t+5} = P^{t+10} = L^{2} * P^{t}$$

etc.

Continued

Population Projection

- To project the total population at a later date
  - First, use the female projection and fertility rates for both boy and girl births
  - Then, put the male births into male projection
  - Note: The male matrix only contains survival ratios

Projection Adjusting for Net Migration (NM)

$$P^{t+5} = L * P^{t} + NM$$

- Where NM = Net migration
- It is easiest to take net migration into account at the end of each time interval
- Getting estimates of net migration by age and sex, and projected numbers by age and sex, may be difficult

Projection Adjusting for Changes in Fertility and Mortality

- Let L^i = Projection matrix for i^{th} interval

  L^0 = Projection matrix for first interval

Population Projection

- Ex: At time t+10

$$P^{t+10} = L^{1} * \left(L^{0} * P^{t} \right)$$

- Or more generally

$$P^{t+5*n} = L^{n-1} * L^{n-2} \ldots * L^{0} * P^{t}$$

- Where L^i = Projection matrix for i^{th} time interval

Continued

Population Projection

- Note: Stable population results from unchanging rates of births and deaths
- Since L_n "stabilizes" when "n" gets large (i.e., for every element), its value in "L^{n+1}" divided by its value in "L^n" converges to a constant "λ"

Continued

Population Projection

- Therefore:

$$L^{n+1} * P^{t} = \lambda * L^{n} * P^{t}$$

$$\text{so } \lambda * P^{t} = P^{t+5}, \text{ and}$$

$$\lambda = \frac{P^{t+5}}{P^{t}} = e^{5r} \quad \text{in continuous notation}$$

Continued

Population Projection

- Several variations of projection exist for changing rates
  - Change fertility rates by x% and/or mortality rates by y%, at each step
  - Fix final levels of fertility and mortality and interpolate between the two time periods
  - Note: Net migration can also be easily made to vary

Projection for Small Areas

- Internal migration (e.g., rural-urban migration) may be important
- Population projection for political units with relatively few boundary changes (e.g., states) and for statistical subdivisions with boundaries changing (e.g., urban/rural) are two different problems

Ratio-Correlation or Regression Methods

- Define:

$$Y_i = \frac{\text{percent of total population n area i, time 1}}{\text{percent of total population n area i, time 0}}$$

- Write equation (can use different number of Xs):

$$Y_i = \alpha + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots$$

Projection for Small Areas

- For example:

$$X_{i1} = \frac{\text{percent of total births area i, time 1}}{\text{percent of total births area i, time 0}}$$

$$X_{i2} = \frac{\text{percent of total school enrollees area i, time 1}}{\text{percent of total school enrollees area i, time 0}}$$

Continued

Projection for Small Areas

- Estimate α, β_1, β_2 statistically, and use these to predict Y_i for some future time t2
  - Note: Y_i is a percent and if all areas are done, the percentages may not add to 100
- So, they must be adjusted to 100 and then multiplied by the estimated total population at t2 to get the projection for area “i” at that time

Administrative Records

$$P_t = H_t \text{ x } PPH_t + GQ_t$$

- P_t = Total population at time "t"
  H_t = Occupied housing units at time "t"
  PPH_t = Average number of persons per household at time "t"
  GQ_t = Population in group quarters at time "t"

Combined Methods

- Average of several methods
- Method used by the U.S. Census Bureau

Judging Projections

- The evaluation of projections requires some standard by which to judge their quality
- One may reasonably compare a projection with the population actually recorded later using percent differences to indicate how far it deviates from the actual figure

Continued

Judging Projections

- The concept of "accuracy" becomes less meaningful where several series of projections are offered as reasonable possibilities, and particularly where none is offered as a "forecast"
- The United Nations uses high, medium, low, and constant (unchanging fertility and mortality) projections

Continued

Judging Projections

- Where possible, it may be more profitable to compare the actual components of population change (i.e., births, deaths, and net migration) with their projected figures because it provides insight into the reasonableness of the various assumptions

Proportional Error

- Hindsight, calculate the proportional error:

$$\text{Proportional error} = \frac{P_{actual} - P_{proj}}{P_{actual}}$$

- Where P_{actual} = Actual population (e.g., census count at time "t")

  P_{proj} = Population projection for time "t"

Population Projection

- One can also assess projections by looking at the width of the range from the highest and the lowest series in a set of principal projections

Continued

Population Projection

- This width of the range depends on the regularity of the following:
  - Past demographic trends, knowledge regarding past trends
  - Ability to measure them accurately
  - Analyst's judgment of the likely course of future change

Continued

Population Projection

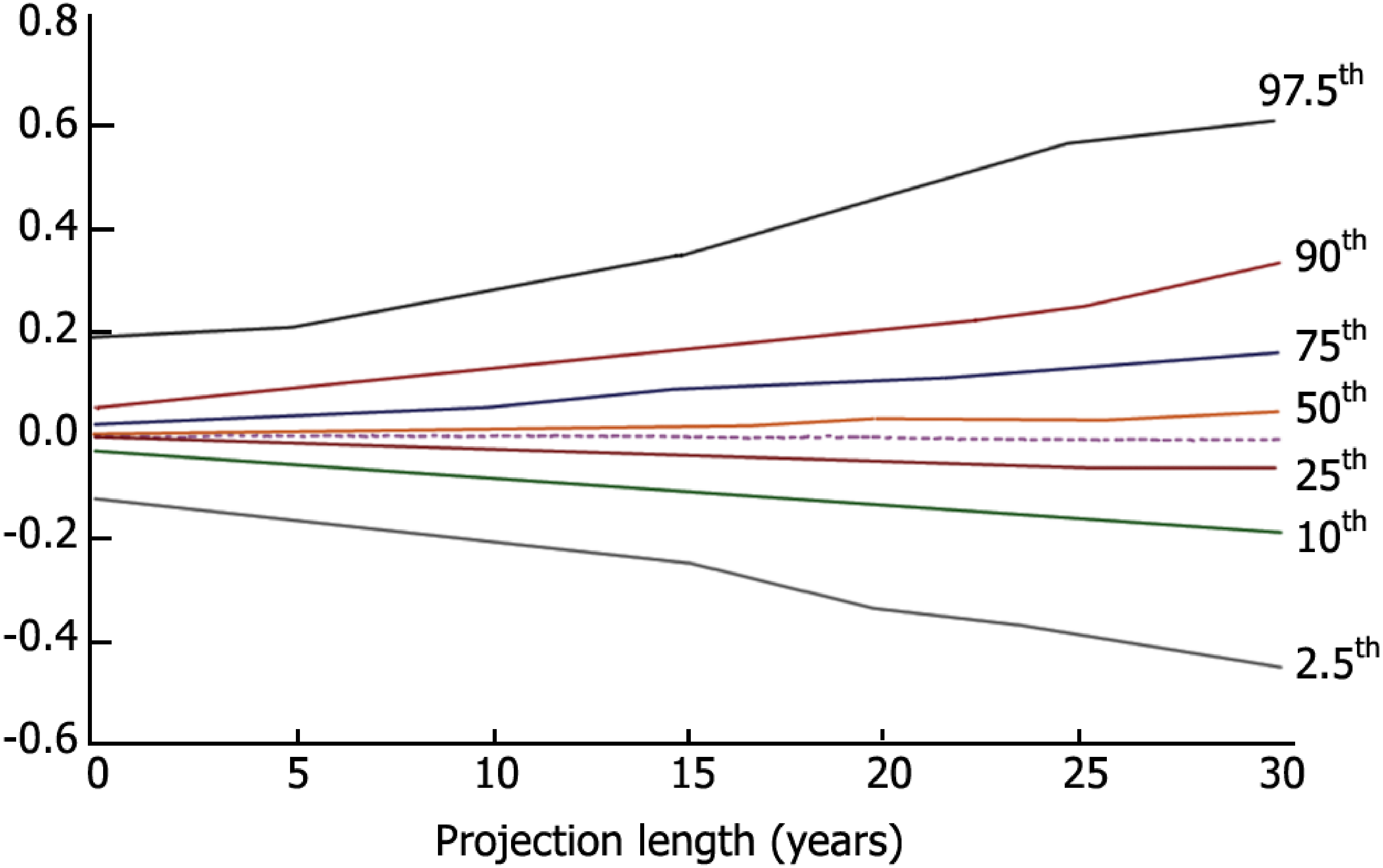
- As the range widens, the analyst is indicating that he/she has less and less confidence in the projections

Error in Projected Population by Target Year: Means across Countries and Forecasts

Target year	Proportional error	Absolute proportional error	(n)
1975	-0.004	0.062	163
1980	0.009	0.076	439
1985	0.010	0.069	635
1990	0.016	0.080	1,194
1995	0.025	0.089	1,138
2000	0.036	0.109	1,375

Adapted from John Bongaarts and Rodolfo A. Bulatao, Editors, Beyond six billion, National Academy Press, 2000

Proportional Error by Projection Length: Percentile for All Countries



Source: John Bongaarts and Rodolfo A. Bulatao, Editors, Beyond six billion, National Academy Press, *2000*

Uses of Projections

- Forecasting future population change
- Warning of population increase or decrease
- Sensitivity analyses to see how population will change with changes in assumptions about fertility, mortality, and/or migration

Examples of Applications

- Projection of population by age and sex to estimate the age distribution of the population at a later date
- Projection of children of school-age and apply school enrollment ratios to evaluate the needs in teachers and schools
- Projection for the labor force

Computer Programs

- Examples:
  - DEMPROJ (Futures Group International) to create projections for policy presentations or planning exercises and to produce the inputs required by the other programs of the integrated package SPECTRUM
  - IPSS (SENECIO Software Inc.) for interactively exploring population dynamics, population projection, and the life table program for Macintosh

Population Projection

- Contact:

Director
Population Division, United Nations
United Nations Plaza, Rm. DC2-1950
New York, NY 10017 USA

Fax: (212) 963-2147
Email: population@un.org

Population Information Network (POPIN) web site:
http://www.undp.org/popin

Summary

- Population projection is a forecast of population change using estimates of fertility, mortality, and migration
- No projection is 100 percent accurate, i.e., there is always some degree of uncertainty
- The United Nations uses low, medium, high, and constant projections

Continued

Summary

- Various computer programs have been developed to help the demographer simulate the effects on population size of different rates of fertility and mortality