Fertility and Its Measurement

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Section A

Indicators of Fertility Based on Vital Statistics
Definitions

- *Fecundity*—Physiological capacity to conceive
- *Infecundity (sterility)*—Lack of the capacity to conceive
  - *Primary sterility*—Never able to produce a child
  - *Secondary sterility*—Sterility after one or more children have been born

Continued
**Definitions**

- *Fecundability*—Probability that a woman will conceive during a menstrual cycle
- *Fertility (natality)*—Manifestation of fecundity
- *Infertility*—Inability to bear a live birth
- *Natural fertility*—Fertility in the absence of deliberate parity-specific control
Definitions

- **Reproductivity**—Extent to which a group is replacing its own numbers by natural processes
- **Gravidity**—Number of pregnancies a woman has had
- **Parity**—Number of children born alive to a woman
Definitions

- *Birth interval*—Time between successive live births
- *Pregnancy interval*—Time between successive pregnancies of a woman
Crude Birth Rate (CBR)

- Let $B$ = Number of births
- Let $P$ = Mid-year population
- Let $W_{15-44}$ = Number of women of reproductive ages
Crude Birth Rate (CBR)

Crude Birth Rate—Number of births per 1,000 population

\[
\frac{B}{P} * 1000
\]

\[
= \frac{B}{W_{15-44}} * \frac{W_{15-44}}{P} * 1000
\]
Exercise

**Crude Birth Rate (CBR)**

- Use the following data to calculate the CBR per 1,000

**Island of Mauritius, 1985**

<table>
<thead>
<tr>
<th>Total Births: 18,247</th>
<th>Total female population: 491,310</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total male population: 493,900</td>
<td></td>
</tr>
</tbody>
</table>

*You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.*

*Source: U.N. Demographic Yearbook, 1986*
Exercise Answer

**Crude Birth Rate (CBR)**

- The correct answer is as follows:
  - **18.5 births per 1,000 population**

---

**Island of Mauritius, 1985**

- Total Births: 18,247
- Total female population: 491,310
- Total male population: 493,900
Crude Birth Rate (CBR)

- Crude birth rates can be standardized using the direct or the indirect method
- Example: Direct (DSBR) and indirect (ISBR) standardization of the Island of Mauritius (I.M.) 1985 crude birth rate using Mali's 1987 data as standard
## Direct Standardization of Birth Rate for Mauritius Island

<table>
<thead>
<tr>
<th>Age group</th>
<th>(Study) Rates I.M. per 1000</th>
<th>(Standard) Population Mali</th>
<th>Expected number of births, I.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>18</td>
<td>725719</td>
<td>13063</td>
</tr>
<tr>
<td>20-24</td>
<td>58</td>
<td>574357</td>
<td>33313</td>
</tr>
<tr>
<td>25-29</td>
<td>57</td>
<td>536226</td>
<td>30565</td>
</tr>
<tr>
<td>30-34</td>
<td>36</td>
<td>443702</td>
<td>15973</td>
</tr>
<tr>
<td>35-39</td>
<td>19</td>
<td>379184</td>
<td>7204</td>
</tr>
<tr>
<td>40-44</td>
<td>6</td>
<td>325824</td>
<td>1955</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>2985012</td>
<td>102073</td>
</tr>
</tbody>
</table>

Total number of births, Mali: 375117  
Total number of births, I.M.: 18247

CBR Mali: 48.7  
CBR I.M. 18.5

### Indirect Standardization of Birth Rate for Mauritius Island

<table>
<thead>
<tr>
<th>Age group</th>
<th>(Standard) Rates Mali per 1000</th>
<th>(Study) Population I.M.</th>
<th>Expected number of births, I.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>79</td>
<td>105764</td>
<td>8355</td>
</tr>
<tr>
<td>20-24</td>
<td>159</td>
<td>109914</td>
<td>17476</td>
</tr>
<tr>
<td>25-29</td>
<td>171</td>
<td>94576</td>
<td>16172</td>
</tr>
<tr>
<td>30-34</td>
<td>140</td>
<td>81144</td>
<td>11360</td>
</tr>
<tr>
<td>35-39</td>
<td>107</td>
<td>60063</td>
<td>6427</td>
</tr>
<tr>
<td>40-44</td>
<td>50</td>
<td>45825</td>
<td>2291</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>497286</td>
<td>62082</td>
</tr>
</tbody>
</table>

- Total number of births, Mali: 375117
- Total number of births, I.M.: 18247

- CBR Mali: 48.7
- CBR I.M.: 18.5

\[
\text{DSBR I.M.} = \frac{\text{Expected births in I.M.}}{\text{Actual births in Mali}} \times \text{CBR}_{\text{Mali}}
\]
\[
= \frac{102073}{375117} \times 48.7 = 13.3
\]

\[
\text{ISBR I.M.} = \frac{\text{Observed births in I.M.}}{\text{Expected births in I.M.}} \times \text{CBR}_{\text{Mali}}
\]
\[
= \frac{18247}{62082} \times 48.7 = 14.3
\]

\[
\text{CBR I.M.} = 18.5
\]

General Fertility Rate (GFR)

- General Fertility Rate—Number of births per 1,000 women of reproductive ages

\[
\text{GFR} \approx 4 \times \text{CBR}
\]
Exercise
General Fertility Rate (GFR)

Use the following data to calculate the GFR per 1,000 women aged 15–44:

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>52 013</td>
</tr>
<tr>
<td>20-24</td>
<td>54 307</td>
</tr>
<tr>
<td>25-29</td>
<td>46 990</td>
</tr>
<tr>
<td>30-34</td>
<td>40 211</td>
</tr>
<tr>
<td>35-39</td>
<td>30 401</td>
</tr>
<tr>
<td>40-44</td>
<td>23 496</td>
</tr>
</tbody>
</table>

Total births: 18 247

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Source: U.N. Demographic Yearbook, 1986
Exercise

General Fertility Rate (GFR)

- The correct answer is:
  - 73.7 births per 1,000 women aged 15-44

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>52 013</td>
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<tr>
<td>35-39</td>
<td>30 401</td>
</tr>
<tr>
<td>40-44</td>
<td>23 496</td>
</tr>
</tbody>
</table>

Total births: 18 247
Age-Specific Fertility Rate (ASFR)

Let $B_a = \text{Number of births to women of age (group) } \text{“a”}$

$W_a = \text{Number of women of age (group) } \text{“a”}$

$n = \text{Number of years in age group}$
Age-Specific Fertility Rate (ASFR\(a, n\))

- \(\text{ASFR}(a, n)\) — Number of births per 1,000 women of a specific age (group)

\[
\text{F}_a = \frac{B_a}{W_a} \times 1000
\]

- If \(n = 1\), then write ASFR\((a)\)
- Example: ASFR Poland 1984
Age-Specific Fertility Rates
Poland, 1984

Age Group
Rate (per thousand)
0 50 100 150 200
Exercise

Age-Specific Fertility Rate ($ASFR[a, n]$)

Use the following data to calculate the ASFR per 1,000 for women age 20–24 and 25–29

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Births</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>52 013</td>
<td>1884</td>
</tr>
<tr>
<td>20-24</td>
<td>54 307</td>
<td>6371</td>
</tr>
<tr>
<td>25-29</td>
<td>46 990</td>
<td>5362</td>
</tr>
<tr>
<td>30-34</td>
<td>40 211</td>
<td>2901</td>
</tr>
<tr>
<td>35-39</td>
<td>30 401</td>
<td>1170</td>
</tr>
<tr>
<td>40-44</td>
<td>23 496</td>
<td>268</td>
</tr>
</tbody>
</table>

*Source: U.N. Demographic Yearbook, 1986*
Exercise Answer

Age-Specific Fertility Rate (ASFR)

The correct answers are:
- **ASFR(20,5) = 117.3 births per 1,000 women 20–24**
- **ASFR(25,5) = 114.1 births per 1,000 women 25–29**

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Births</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
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</tr>
</tbody>
</table>
Total Fertility Rate (TFR)

- *Total Fertility Rate*—Number of children a woman will have if she lives through all the reproductive ages and follows the age-specific fertility rates of a given time period (usually one year)
Total Fertility Rate (TFR)

- For single-year age groups

\[
TFR = \sum_{a=15}^{44} \frac{B_a}{W_a} \times 1000 = \sum \text{ASFR}(a) = \sum F_a
\]

- For five-year age groups

\[
TFR = 5 \times \sum_{a=15-19}^{40-44} \frac{B_a}{W_a} \times 1000 = 5 \times \sum \text{ASFR}(a,5)
\]
Total Fertility Rate (TFR)

Example: ASFR and TFR—Poland, 1984

<table>
<thead>
<tr>
<th>Age group</th>
<th>Ba</th>
<th>Wa</th>
<th>ASFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>43807</td>
<td>1230396</td>
<td>35.60</td>
</tr>
<tr>
<td>20-24</td>
<td>257872</td>
<td>1390077</td>
<td>185.51</td>
</tr>
<tr>
<td>25-29</td>
<td>236088</td>
<td>1653183</td>
<td>142.81</td>
</tr>
<tr>
<td>30-34</td>
<td>115566</td>
<td>1608925</td>
<td>71.83</td>
</tr>
<tr>
<td>35-39</td>
<td>38450</td>
<td>1241967</td>
<td>30.96</td>
</tr>
<tr>
<td>40-44</td>
<td>6627</td>
<td>941963</td>
<td>7.04</td>
</tr>
</tbody>
</table>

TFR = \( \frac{5 \times 473.74}{1000} = 2.4 \)
Exercise

Total Fertility Rate (TFR)

Use the following data to calculate the TFR per 1,000

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<thead>
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<tbody>
<tr>
<td>15–19</td>
<td>52,013</td>
<td>1,884</td>
</tr>
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<td>20–24</td>
<td>54,307</td>
<td>6,371</td>
</tr>
<tr>
<td>25–29</td>
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<td>40,211</td>
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<td>35–39</td>
<td>30,401</td>
<td>1,170</td>
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Source: U.N. Demographic Yearbook, 1986
The correct answer is as follows:

- **TFR = 1.9 children per woman**

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</table>
Mean of Age of Childbearing

- For single-year groups

\[
\bar{a} = \frac{\sum_{a=15}^{44} (a + 1/2) F_a}{\sum_{a=15}^{44} F_a}
\]

- For five-year age groups

\[
\bar{a} = \frac{\sum (a + 2.5) F_a}{\sum F_a}
\]
Variance of Age of Childbearing

\[ s^2 = \frac{\sum_{a=15}^{44} \left( a - \bar{a} \right)^2 \cdot F_a}{\sum_{a=15}^{44} F_a} \]
Exercise

Mean and Variance of Age of Childbearing

Use the following data to calculate the mean and variance of age of childbearing

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<td>23,496</td>
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</tr>
</tbody>
</table>

Source: U.N. Demographic Yearbook, 1986
Exercise Answer
Mean and Variance of Age of Childbearing

The correct answers are as follows:
- Mean age of childbearing = 27.9 years
- Variance of age of childbearing = 38.2 years

<table>
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<td>268</td>
</tr>
</tbody>
</table>
Mean and Median Age of Mothers

Mean:

\[
\alpha \sum_{a=\alpha}^{\beta} \left( a + \frac{1}{2} \right) B_a \sum B_a
\]

Median \( x \) such that:

\[
\frac{1}{44} \sum_{a=15}^{15} B_a = 0.5
\]

\( B_a = \text{Number of births to women age } a \)
Exercise

Mean and Median Age of Mothers

Use the following data to calculate the mean and median age of mothers.

**Island of Mauritius, 1985**

<table>
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<tr>
<th>Age Group of Mother</th>
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</tr>
</thead>
<tbody>
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<td>40–44</td>
<td>23,496</td>
<td>268</td>
</tr>
</tbody>
</table>

Source: U.N. Demographic Yearbook, 1986
Exercise Answer
Mean and Median Age of Mothers

- The correct answers are as follows:
  - Mean age of mothers = 26.9 years
  - Median age of mothers = 24.7 years

<table>
<thead>
<tr>
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<th>Births</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1,170</td>
</tr>
<tr>
<td>40-44</td>
<td>23,496</td>
<td>268</td>
</tr>
</tbody>
</table>
Parity

- **Mean**

\[ l = \max(i) \]
\[ \sum_{i=1}^{l} M_i \]

- **Median** such that:

\[ x \]
\[ \sum_{i=1}^{l} m_i = 0.5 \]

- \( M_i = \text{Proportion of women at or above parity } i \)

- \( m_i = \text{Proportion of women at parity } i \)
Marital Fertility Rate (MFR)

- Let $B_{m} = \text{Number of marital births}$
- $B_{u} = \text{Number of non-marital births}$
- $W_{m}^{15-44} = \text{Number of married women of age 15–44}$
- $W_{u}^{15-44} = \text{Number of unmarried women of age 15–44}$
General Marital Fertility Rate (GMFR)

- *General Marital Fertility Rate*—Number of births per 1,000 married women of reproductive ages

\[
\frac{B}{W_{15-44}^m} \times 1000
\]
Marital Fertility Rate (MFR)

- *Marital Fertility Rate*—Number of marital births per 1,000 married women of reproductive ages

\[
B_m = \frac{B_m}{W_{15-44}^m} \times 1000
\]

39
“Out-of-Wedlock” (Non-Marital) Fertility Rate

“Out-of-Wedlock” (Non-Marital) Fertility Rate—Number of non-marital births per 1,000 unmarried women of reproductive ages

\[
\frac{B_u}{W_{15-44}^u} \times 1000
\]
Some Relationships

\[ \frac{B_m}{W^m_{15-44}} + \frac{B_u}{W^u_{15-44}} \neq \frac{B}{W_{15-44}} \]

- But

\[ \frac{B_m}{W^m_{15-44}} \ast \frac{W^m_{15-44}}{W_{15-44}} + \frac{B_u}{W^u_{15-44}} \ast \frac{W^u_{15-44}}{W_{15-44}} = \frac{B}{W_{15-44}} \]
Age-Specific Marital Fertility Rate (ASMFR)

- Let $B^m_a = \text{Number of marital births to women of age group \"a\"}$
- $W^m_a = \text{Number of married women in age group \"a\"}$
- $B^m_{(d)} = \text{Number of marital births to women married for \"d\" years}$
- $W^m_{(d)} = \text{Number of women married for \"d\" years}$
Age-Specific Marital Fertility Rate (ASMFR)

- *Age-Specific Marital Fertility Rate*—Number of marital births per 1,000 married women of age (group) “a”

\[
\text{B}_{mla} \div \text{W}_{ma} \times 1000
\]
Duration (of Marriage)—Specific Fertility Rate (DSFR)

- **DSFR**—Number of marital births per 1,000 women who have been married for duration “d”

\[
\text{DSFR} = \frac{B_l(d)}{W^m(d)} \times 1000
\]
Order-Specific Fertility Rate (OSFR)

- Let $B^i = \text{Number of births of order } \text{"i"}, \ i > 0$
- $B^i_{a} = \text{Number of order } \text{"i"} \text{ births to women in age group } \text{"a"}$
- $W_{a} = \text{Number of women in age group } \text{"a"}$
- $W_{15-44} = \text{Number of women of age 15–44 (or 15–49)}$
Order-Specific Fertility Rate (OSFR)

- *Order-Specific Fertility Rate*—Number of order “i” births per 1,000 women of reproductive ages

\[
\frac{B^i}{W_{15-44}} \times 1000
\]

- Example: OSFR Poland 1984
Order-Specific Fertility Rates
Poland, 1984

Rate (per thousand)

Birth Order
**Exercise**

*Order-Specific Fertility Rate (OSFR)*

- Use the following data to calculate the OSFR for birth orders 1 and 3

<table>
<thead>
<tr>
<th>Island of Mauritius, 1985</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Group of Mother of Mother</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>40</td>
</tr>
</tbody>
</table>

*Source:* U.N. Demographic Yearbook, *1986*
Exercise Answer

Order-Specific Fertility Rate (OSFR)

- The correct answers are as follows:
  - \( \text{OSFR}(1) = 28.8 \) births of order 1 per 1,000 women 15-44
  - \( \text{OSFR}(3) = 11.2 \) births of order 3 per 1,000 women 15-44

<table>
<thead>
<tr>
<th>Island of Mauritius, 1985</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Group of Mother</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>15–19</td>
</tr>
<tr>
<td>20–24</td>
</tr>
<tr>
<td>25–29</td>
</tr>
<tr>
<td>30–34</td>
</tr>
<tr>
<td>35–39</td>
</tr>
<tr>
<td>40–44</td>
</tr>
</tbody>
</table>
Age-Order Specific Fertility Rate (AOSFR [{a,i}])

- \( AOSFR(a, i) \) — Number of order “i” births per 1,000 women of age (group) “a”

\[
\frac{B^i_a}{W_a} \times 1000
\]

- Example: AOSFR Poland 1984
Age-Order Specific Fertility Rates
Poland 1984

- 1st birth
- 2nd
- 3rd
- 4th +
Exercise

Age-Order Specific Fertility Rate (AOSFR)

- Use the following data to calculate the AOSFR for birth order 3 in age group 25–29

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Birth order 1</th>
<th>Birth order 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>15919</td>
<td>52,013</td>
<td>1,521</td>
<td>40</td>
</tr>
<tr>
<td>20924</td>
<td>54,307</td>
<td>3,317</td>
<td>678</td>
</tr>
<tr>
<td>25929</td>
<td>46,990</td>
<td>1,638</td>
<td>1,132</td>
</tr>
<tr>
<td>30934</td>
<td>40,211</td>
<td>496</td>
<td>665</td>
</tr>
<tr>
<td>35939</td>
<td>30,401</td>
<td>142</td>
<td>215</td>
</tr>
<tr>
<td>40944</td>
<td>23,496</td>
<td>24</td>
<td>30</td>
</tr>
</tbody>
</table>

Source: U.N. Demographic Yearbook, 1986
Exercise Answer

Age-Order Specific Fertility Rate (AOSFR)

- The correct answer is as follows:
  - \( \text{AOSFR} = 24.1 \text{ births of order 3 per 1,000 women 25-29} \)

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Birth order 1</th>
<th>Birth order 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>15–19</td>
<td>52,013</td>
<td>1,521</td>
<td>40</td>
</tr>
<tr>
<td>20–24</td>
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</tr>
<tr>
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<td>215</td>
</tr>
<tr>
<td>40–44</td>
<td>23,496</td>
<td>24</td>
<td>30</td>
</tr>
</tbody>
</table>
Age-Order Specific Fertility Rate (AOSFR[a,i])

Note:

\[
\sum_{i=1}^{\infty} \text{AOSFR} (a,i) = \text{ASFR} (a)
\]

\[
\sum_{a=15}^{44} \text{AOSFR} (a,i) \times \frac{W_a}{W_{15-44}} = \text{OSFR}(i)
\]
Cumulative Order Specific Birth Rate (COSFR\([i,a]\))

- Cumulative up to age “\(a\)”
- \(COSFR(i,a)\)—Total number of order “\(i\)” births per 1,000 women of age (group) less than or equal to “\(a\)”

\[
=a \sum_{x=0}^{a} AOSFR(i,x)
\]
Birth Probability

- Let $B^i$ = Number of births of order “i”, $i > 0$, in year “t”
- $W^{i-1}$ = Number of women of parity “i-1” at beginning of year “t”
Birth Probability, BP(i)

- *Birth Probability*—Probability of having an “ith” order birth in a given year for women who already have “i-1” births

\[
BP(i) = \frac{B^i}{W^{i-1}}
\]
Birth Probability

- May be age-specific as well
- Birth probabilities are the most sensitive indicators of temporal change in the pace of childbearing
Paternal Fertility Rate

- Let $B$ = Number of births
  
  $M_{15-54}$ = Number of men of age 15–54
General Fertility Rate of Men (GFR_m)

- General Fertility Rate of Men—Number of births per 1,000 men age 15 to 54

\[
B = \frac{M_{15-54}}{1000}
\]
Summary

- Fertility data are collected from vital statistics, censuses, or surveys.
- Vital statistics principally provide birth statistics.
- Many indicators have been developed to understand and explain the fertility patterns in populations based on vital statistics.
Section B

Indicators of Reproduction Based on Vital Statistics
Gross Reproduction Rate (GRR)

- Let $B^f$ = Number of female births
- $B^{m+f}$ = Number of male and female births, i.e., all births
Gross Reproduction Rate (GRR)

- *Gross Reproduction Rate*—Number of daughters expected to be born alive to a hypothetical cohort of women (usually 1,000) if no one dies during childbearing years and if the same schedule of age-specific rates is applied throughout the childbearing years.
If the sex ratio at birth is assumed constant across ages of women
Exercise

Gross Reproduction Rate (GRR)

Use the following data to calculate the GRR

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Total</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>8 651</td>
<td>522</td>
<td>267</td>
</tr>
<tr>
<td>20-24</td>
<td>9 345</td>
<td>094</td>
<td>560</td>
</tr>
<tr>
<td>25-29</td>
<td>10 617</td>
<td>1277</td>
<td>653</td>
</tr>
<tr>
<td>30-34</td>
<td>10 986</td>
<td>886</td>
<td>454</td>
</tr>
<tr>
<td>35-39</td>
<td>10 061</td>
<td>318</td>
<td>163</td>
</tr>
<tr>
<td>40-44</td>
<td>8 924</td>
<td>49</td>
<td>25</td>
</tr>
</tbody>
</table>

Numbers are in 1,000s

You have 15 seconds to calculate the answer. You may pause the presentation if you need more time.

Exercise Answer

Gross Reproduction Rate (GRR)

- The correct answer for the gross reproduction rate is as follows:
  - 1.01 daughters per woman

<table>
<thead>
<tr>
<th>Age Group of Births</th>
<th>Mother</th>
<th>Women</th>
<th>Total</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>8 651</td>
<td>522</td>
<td>267</td>
<td></td>
</tr>
<tr>
<td>20-24</td>
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</tr>
<tr>
<td>40-44</td>
<td>8 924</td>
<td>49</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

Numbers are in 1,000s

United States, 1990
Net Reproduction Rate (NRR)

- Let $L^f_a = \text{Life table person-years lived by women in age group “a”}$
- $l_0 = \text{Radix of life table}$
- $B^f = \text{Number of female births}$
- $B^{m+f} = \text{Number of male and female births}$
Net Reproduction Rate (NRR)

- *Net Reproduction Rate*—Average number of daughters expected to be born alive to a hypothetical cohort of women if the same schedule of age-specific fertility and mortality rates applied throughout the childbearing years.
Net Reproduction Rate (NRR)

- For single-year age groups

\[ \text{NRR} = \sum \text{ASFR} \times \frac{1}{L_a^f} \times \frac{B^f}{B^{m+f}} \]

- For five-year age groups

\[ \text{NRR} = \sum \text{ASFR} \times \frac{5}{L_a^f} \times \frac{B^f}{B^{m+f}} \]
Exercise

Net Reproduction Rate (NRR)

Use the following data to calculate the NRR

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Total</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>8,651</td>
<td>522</td>
<td>267</td>
</tr>
<tr>
<td>20-24</td>
<td>9,345</td>
<td>094</td>
<td>560</td>
</tr>
<tr>
<td>25-29</td>
<td>10,617</td>
<td>1277</td>
<td>653</td>
</tr>
<tr>
<td>30-34</td>
<td>10,986</td>
<td>886</td>
<td>454</td>
</tr>
<tr>
<td>35-39</td>
<td>10,061</td>
<td>318</td>
<td>163</td>
</tr>
<tr>
<td>40-44</td>
<td>8,924</td>
<td>49</td>
<td>25</td>
</tr>
</tbody>
</table>

Numbers are in 1,000s

The correct answer for the NRR is as follows:

- **1.00 daughter per woman**

### United States, 1990

<table>
<thead>
<tr>
<th>Age Group of Mother</th>
<th>Women</th>
<th>Total Births</th>
<th>Males</th>
<th>Stationary Population &lt;sup&gt;5&lt;/sup&gt; &lt;sub&gt;L_x&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>8 651</td>
<td>522</td>
<td>267</td>
<td>493 629</td>
</tr>
<tr>
<td>20-24</td>
<td>9 345</td>
<td>094</td>
<td>560</td>
<td>492 399</td>
</tr>
<tr>
<td>25-29</td>
<td>10 617</td>
<td>1277</td>
<td>653</td>
<td>490 989</td>
</tr>
<tr>
<td>30-34</td>
<td>10 986</td>
<td>886</td>
<td>454</td>
<td>489 203</td>
</tr>
<tr>
<td>35-39</td>
<td>10 061</td>
<td>318</td>
<td>163</td>
<td>486 812</td>
</tr>
<tr>
<td>40-44</td>
<td>8 924</td>
<td>49</td>
<td>25</td>
<td>483 465</td>
</tr>
</tbody>
</table>

Numbers are in 1,000s
Net Reproduction Rate (NRR)

\[ \text{NRR} \approx \ell \left( \bar{\alpha} \right) \times \text{GRR} \]

- Where \( \ell(\bar{\alpha}) = \text{Life table probability of surviving beyond } \bar{\alpha} \)
  - \( \bar{\alpha} = \text{mean age of childbearing} \) usually \( 20 < \bar{\alpha} < 30 \)

Completed Fertility Rates for Birth Cohorts of Women

Reproductivity

- Reproductivity is usually studied in terms of mothers and daughters because of the following reasons:
  - The fecund period for females is shorter than it is for males
  - Characteristics such as age are much more likely to be known for the mothers of illegitimate babies than for their fathers
Summary

- Reproductivity data are collected from vital statistics
- Many indicators have been developed to understand and explain reproductivity patterns in populations
Section C

Indicators of Fertility Based on Censuses and Surveys
Child-Woman Ratio (CWR)

- *Child-Woman Ratio*—Number of children zero to four years old relative to number of women of reproductive ages
Child-Woman Ratio (CWR)

\[
\text{CWR} = \frac{P_{0-4}}{W_{15-44}} \times 1000
\]

- Where \( P_{0-4} \) = Mid-year population of persons age 0-4 years
- \( W_{15-44} \) = Number of women of reproductive ages
Exercise

Child-Woman Ratio (CWR)

Use the following data to calculate the CWR

Household population composition by age and sex, Kenya 1998

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>268873</td>
<td>256705</td>
<td>525578</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15-19</td>
<td>189272</td>
<td>192067</td>
<td>381340</td>
</tr>
<tr>
<td>20-24</td>
<td>130899</td>
<td>158825</td>
<td>289723</td>
</tr>
<tr>
<td>25-29</td>
<td>116747</td>
<td>142204</td>
<td>258951</td>
</tr>
<tr>
<td>30-34</td>
<td>100827</td>
<td>99727</td>
<td>200555</td>
</tr>
<tr>
<td>35-39</td>
<td>88445</td>
<td>103421</td>
<td>191866</td>
</tr>
<tr>
<td>40-44</td>
<td>67218</td>
<td>68332</td>
<td>135550</td>
</tr>
</tbody>
</table>

Source: Demographic and Health Survey, Kenya 1998
The correct answer is as follows:

- 687.4 children 0-4 per 1,000 women 15-44

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>268873</td>
<td>256705</td>
<td>525578</td>
</tr>
<tr>
<td>...</td>
<td></td>
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</tr>
<tr>
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</tr>
<tr>
<td>40-44</td>
<td>67218</td>
<td>68332</td>
<td>135550</td>
</tr>
</tbody>
</table>
Children Ever Born (CEB)

- *Children Ever Born*—Total number of children a woman has ever given birth to
- The survival status of the children is not considered here
- Almost all censuses tabulate mean CEB by marital status and by age of the mother
Parity Progression Ratios (PPR[i])

- Let $N$ = Random variable for number of births
  
  $m_i$ = Proportion of women of parity “i”
  
  $M_i$ = Proportion of women at or above parity “i”
Parity Progression Ratios (PPR[i])

- *Parity Progression Ratios*—Probability that a woman has an (i+1st) birth given that she already has had “i” births

\[
= a_i \\
= \frac{M_i + 1}{M_i} \\
= \text{Prob} (N \geq i + 1 | N \geq i)
\]
Exercise

Parity Progression Ratios (PPR)

Given the following percentages of women at parity “i” for women 45-49 in 1995 in Colombia, calculate PPR(2)

<table>
<thead>
<tr>
<th>Parity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.2</td>
</tr>
<tr>
<td>1</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>13.9</td>
</tr>
<tr>
<td>3</td>
<td>18.6</td>
</tr>
<tr>
<td>4</td>
<td>15.6</td>
</tr>
<tr>
<td>5+</td>
<td>35.2</td>
</tr>
</tbody>
</table>

Source: Demographic and Health Survey, Colombia 1995
The correct answer for the PPR(2) is as follows:

- **0.83** (i.e., there is an 83% chance that a woman 45–49 has a second birth given she already has had a first birth)

<table>
<thead>
<tr>
<th>Parity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.2</td>
</tr>
<tr>
<td>1</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>13.9</td>
</tr>
<tr>
<td>3</td>
<td>18.6</td>
</tr>
<tr>
<td>4</td>
<td>15.6</td>
</tr>
<tr>
<td>5+</td>
<td>35.2</td>
</tr>
</tbody>
</table>
Parity Progression Ratios (PPR)

- Note: $a_0 = \text{Prob (ever give birth)}$
  
  $1 - a_0 = \text{Prob (never have a birth)}$
  
  $M_0 = 1$

- $a_i$ need not decrease with increasing $i$

- $M_i$ does decrease
Proportion of Women of Parity \( i(mi) \)

- Is an unconditional probability

\[
= M_i - M_{i+1} \\
= \text{Prob} (N \geq i) - \text{Prob} (N \geq i+1) \\
= a_0 \times a_1 \times \ldots \times a_{i-1} \times (1 - a_i)
\]
Proportion of Women of Parity $i(m_i)$

Given the following percentages of women at parity “$i$” for women 45–49 in 1995 in Colombia, verify the relationships indicated on the previous slide:

<table>
<thead>
<tr>
<th>Parity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>4</td>
<td>15.6</td>
</tr>
<tr>
<td>5+</td>
<td>35.2</td>
</tr>
</tbody>
</table>

Source: Demographic and Health Survey, Colombia 1995
Mean Parity

\[
\begin{align*}
\max(i) &= \sum_{i=1}^{\max(i)} M_i = \sum_{i=1}^{\max(i)} i \cdot m_i
\end{align*}
\]
Exercise
Mean Parity

Given the following percentages of women at parity “i” for women 45–49 in 1995 in Colombia, calculate the mean parity using both formulas on the previous slide (assume that women at parity 10+ are on average at parity 11)

<table>
<thead>
<tr>
<th>Parity</th>
<th>Percentage</th>
<th>Parity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.2</td>
<td>6</td>
<td>8.0</td>
</tr>
<tr>
<td>1</td>
<td>7.5</td>
<td>7</td>
<td>5.9</td>
</tr>
<tr>
<td>2</td>
<td>13.9</td>
<td>8</td>
<td>3.2</td>
</tr>
<tr>
<td>3</td>
<td>18.6</td>
<td>9</td>
<td>3.2</td>
</tr>
<tr>
<td>4</td>
<td>15.6</td>
<td>10+</td>
<td>4.8</td>
</tr>
<tr>
<td>5</td>
<td>10.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Demographic and Health Survey, Colombia 1995
Exercise Answer

Mean Parity

- The correct answer is as follows:
  - Mean parity = 4.01

<table>
<thead>
<tr>
<th>Parity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.2</td>
</tr>
<tr>
<td>1</td>
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</tr>
<tr>
<td>4</td>
<td>15.6</td>
</tr>
<tr>
<td>5</td>
<td>10.1</td>
</tr>
</tbody>
</table>

<table>
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<th>Parity</th>
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<td>9</td>
<td>3.2</td>
</tr>
<tr>
<td>10+</td>
<td>4.8</td>
</tr>
</tbody>
</table>
Estimation of GFR from Census Data on Ratio of Children to Women

- Let $P_{0-4}$ = Enumerated number of children under 5
- $W_{15-44}$ = Enumerated number of women of age 15–44
- $W_{20-49}$ = Enumerated number of women of age 20–49
Estimation of GFR from Census Data on Ratio of Children to Women

- Also let $l_0 = \text{Radix of life table}$
- $nL_a = \text{Life table person-years}$
- lived between the ages “$a$” and “$a+n$” (female life table)
Estimation of GFR from Census Data on Ratio of Children to Women

\[
GFR_{est} = \frac{P_{0-4} \times \frac{l_0}{5 \times L_0}}{W_{15-44} + W_{20-49}} \times \frac{30 \times L_{15}}{30 \times L_{20} + 30 \times L_{15}}
\]
Estimation of GFR from Census Data on Ratio of Children to Women

Note:

- Assumes that a life table is available
- Estimated GFR may be used in the absence of birth statistics to compare fertility levels in various areas
Estimation of GFR from Census Data on Ratio of Children to Women

- The life table survivorship functions are inverted in order to estimate the number of persons at the mid-point of the preceding five-year period.
- A lexis diagram makes it easier to understand this calculation.
Summary

- Fertility data are collected from vital statistics, censuses, or surveys

Continued
Summary

- Censuses provide the following:
  - Data on births and fertility
  - Statistics on children by family status of the parents
  - Population data on fertility-related variables
  - Population bases for calculating various types of fertility measures
Summary

- Surveys provide the following:
  - Same type of data as censuses
  - Additional detailed data on special aspects of fertility, including number and timing of births, marriages, pregnancies, birth intervals, and birth interval components
Many indicators have been developed to understand and explain the fertility patterns in populations
Section D

Relationship among Indicators, Indicators and Models of Birth Intervals, and Fertility Models
Relations among Indicators

- At age 50:

\[
\max(i) \sum_{i=1} \text{COSFR}(i,50) = \text{Completed cohort fertility rate} = \text{Mean CEB}
\]

\[
\max(i) \sum_{i=1} M_i
\]
Relations among Indicators

\[
\max(i) \sum_{i=1}^{\text{max}(i)} \text{OSFR}(i) = \max(i) \sum_{i=1}^{\text{max}(i)} \frac{B_i}{W_{15-44}}
\]

\[
= \frac{B}{W_{15-44}}
\]

\[
= \text{GFR}
\]
Pregnancy Histories and Birth Histories

- Data needed
  - Age or birth date of woman
  - Dates of pregnancy terminations
  - Type of termination (live birth or not)
Pregnancy Histories and Birth Histories

- Two ways of collecting data
  - Forward, i.e., from first to last birth
  - Backward, i.e., from last to first birth
Pregnancy Histories and Birth Histories

- Problems
  - Dating of events
  - Forgetting events
  - Age misreporting

Continued
Cohort fertility (births per 1000 women) calculated from survey data for forward and backward questionnaires by five-year age groups and five-year periods before the survey.

Bangladesh, 1984

Legend:

xx: Forward

xx: Backward

Years before the survey

Age

15 20 25 30 35

10 20 30 40 50

112 100 112 317 253

278 277 274 313 332

88 89 110 277 84
Birth Interval Measures and Models

- Let NS = Time from previous pregnancy outcome to resumption of ovulation (postpartum non-susceptibility subinterval)

- For first order births use dates of marriage or beginning of sexual relations
Birth Interval Measures and Models

- Also let $C =$ Time from resumption of ovulation to next conception (conception-wait subinterval)

$G =$ Time from conception to next pregnancy outcome (gestational subinterval)

$W =$ Waiting time due to non-live births
Pregnancy and Live Birth Intervals

- **Pregnancy interval**
  - \( PI = \) Interval between two pregnancies
    \[ = NS + C + G \]

- **Live birth interval**
  - \( LBI = \) Interval between two live births
    \[ = PI + W \]
Observed Birth Intervals

- Closed intervals
- Left censored intervals
- Right censored intervals
- Life table methods can be used to incorporate the different intervals and calculate median birth interval lengths
Birth Interval

Note:

- Mean birth interval for women is different from mean birth interval for births in a given time period
- The latter is shorter
Birth Interval

Mathematical relationships

- In populations with nothing changing
  
  Let \( p \) = Fecundability (assumed fixed)
  
  \( e \) = Effectiveness of contraception
  
  \( u \) = Proportion using contraception
  
  \( s \) = Non-susceptible period (constant)
Mean Birth Interval

Mean birth interval

\[
\frac{1}{\text{Mean Birth Interval}} = \frac{1}{(p (1 - ue))} + s
\]

Fertility rate for fecund women
Mean Conception-Wait Subinterval (MC)

- Mean Conception-Wait Subinterval—Mean time it takes to get pregnant under natural fertility
- Constant fecundability

$$MC = 1 \times p + 2(1 - p)p + 3(1 - p)^2p + \ldots$$

$$= \frac{1}{p}$$

$$p = \text{Monthly probability of conception (assumed fixed in time and for all women)}$$
Heterogeneous Fecundability

- Probability of conception in month “k”
- “p” is assumed to vary between women with distribution “f(p)"

\[
= \int_0^1 p q^{k-1} f(p) \, dp
\]

- Where \( q = 1 - p \)
Probability of Conceiving

In Month “k” Given No Conception Before Then

\[
= 1 - \frac{\int_0^1 q^k f(1 - q) \, dq}{\int_0^1 q^{k-1} f(1 - q) \, dq}
\]

♦ Note

- Here the probability of conception decreases over time
- This has important implications for the study of infertility
Let $r(a)$ = Observed fertility schedule at age “a”

$n(a)$ = Natural fertility pattern at age “a”

$M$ = Level of fertility (estimated)

$m$ = Degree to which fertility control is practiced (estimated)

$v(a)$ = Estimated from U.N. Demographic Yearbook 1965; data for 43 fertility schedules
\[
\frac{r(a)}{n(a)} = M \ast e^{(m \ast v(a))}
\]
Bongaarts model of intermediate fertility variables

Let $C_m =$ Index of marriage

$C_c =$ Index of contraception

$C_a =$ Index of induced abortion

$C_i =$ Index of post-partum infecundability

$K =$ Estimated total natural fertility rate (15.3 [Bongaarts 1982])

Each “C” varies between 0.0 and 1.0
Methods of estimation of each “C” are provided by Bongaarts (1982)

\[ TFR = C_m * C_c * C_a * C_i * K \]
Summary

- Information on women's fertility can be obtained by asking them to report their pregnancy and birth histories.
- Indicators and models have been developed to understand and explain the fertility patterns in populations.