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Section C

The Paired t-Test; Two More Examples
Clinical Agreement by Two Diagnosing Physicians

- Two different physicians assessed the number of palpable lymph nodes in 65 randomly selected male sexual contacts of men with AIDS or AIDS-related conditions.¹

<table>
<thead>
<tr>
<th></th>
<th>Doctor 1</th>
<th>Doctor 2</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ((\bar{X}))</td>
<td>7.91</td>
<td>5.16</td>
<td>-2.75</td>
</tr>
<tr>
<td>sd (s)</td>
<td>4.35</td>
<td>3.93</td>
<td>2.83</td>
</tr>
</tbody>
</table>

95% Confidence Interval

- 95% CI for difference in mean number of lymph nodes, Doctor 2 compared to Doctor 1

\[
\bar{x}_{diff} \pm 2 \times SE(\bar{x}_{diff})
\]

\[
\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{65}}
\]

\[
2.75 \pm 2 \times \left( \frac{2.83}{\sqrt{65}} \right)
\]

- 3.45 to - 2.05
Getting a p-Value

- **Hypotheses**
  - $H_0$: $\mu_{\text{diff}} = 0$
  - $H_A$: $\mu_{\text{diff}} \neq 0$

- First, start by “assuming” null is true and computing distance (in SEs) between $\bar{x}_{\text{diff}}$ and 0

  - Sample result is 7.8 SEs below 0—*is this unusual?*

\[
t = \frac{\bar{x}_{\text{diff}} - 0}{\hat{SE}(\bar{x})} = \frac{-2.75}{2.83/\sqrt{65}} = -7.8
\]
Getting a p-Value

- Sample result is 7.8 SEs below 0—is this unusual?
  - See where this falls on sampling distribution of all possible mean differences based on random samples of 65 patients
    - Theory tells us this is normal

- The p-value is probability of being 7.8 or more standard errors from 0 under a standard normal curve
  - Without looking up, we know p <<< .001!
. ttesti 65 -2.75 2.83 0

One-sample t test

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs</td>
<td>Mean</td>
<td>Std. Err.</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>x</td>
<td>65</td>
<td>-2.75</td>
<td>0.3510183</td>
<td>2.83</td>
</tr>
</tbody>
</table>

mean = mean(x)
Ho: mean = 0                     t = -7.8343
Ha: mean < 0                     degrees of freedom = 64
Pr(T < t) = 0.0000
Ha: mean ≠ 0                     Pr(|T| > |t|) = 0.0000
Pr(T > t) = 1.0000
Ha: mean > 0
**Oat Bran and LDL Cholesterol**

- Cereal and cholesterol: 14 males with high cholesterol given oat bran cereal as part of diet for two weeks, and corn flakes cereal as part of diet for two weeks

<table>
<thead>
<tr>
<th></th>
<th>Corn Flakes</th>
<th>Oat Bran</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($\bar{x}$)</td>
<td>4.44 mmol/dL</td>
<td>4.08</td>
<td>0.36</td>
</tr>
<tr>
<td>sd (s)</td>
<td>1.0</td>
<td>1.1</td>
<td>0.40</td>
</tr>
</tbody>
</table>

95% Confidence Interval

- 95% CI for difference in mean LDL, corn flakes vs. oat bran

\[
\bar{x}_{\text{diff}} \pm t_{0.95,13} \times \text{SE}(\bar{x}_{\text{diff}})
\]

\[
\bar{x}_{\text{diff}} \pm 2 \times \frac{s_{\text{diff}}}{\sqrt{14}}
\]

\[
0.36 \pm 2 \times \left( \frac{0.040}{\sqrt{14}} \right)
\]

0.13 to 0.60 mmol/dL
Getting a p-Value

- Hypotheses
  - $H_0: \mu_{\text{diff}} = 0$
  - $H_A: \mu_{\text{diff}} \neq 0$

- First, start by “assuming” null is true, and computing distance (in SEs) between $\bar{x}_{\text{diff}}$ and 0
  - Sample result is 3.3 SEs above 0—*is this unusual?*

$$t = \frac{\bar{x}_{\text{diff}} - 0}{\hat{SE}(\bar{x})} = \frac{.036}{.04/\sqrt{14}} \approx 3.3$$
Getting a p-Value

- Sample result is 3.3 SEs above 0—is this unusual?
  - See where this falls on sampling distribution of all possible mean differences based on random samples of 14 patients: theory tells us this is $t_{13}$

- The p-value is probability of being 3.3 or more standard errors from 0 under a $t_{13}$ curve: look up in table or go to Stata
. ttesti 14 .36 .40 0

One-sample t test

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<thead>
<tr>
<th></th>
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<th>Std. Err.</th>
<th>Std. Dev.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
<td>.36</td>
<td>.1069045</td>
<td>.40</td>
<td>.1290469 .5909531</td>
</tr>
</tbody>
</table>

mean = mean(x)                                             t = 3.3675
Ho: mean = 0                                               degrees of freedom = 13
Ha: mean < 0                                              Ha: mean != 0
Pr(T < t) = 0.9975                                          Pr(|T| > |t|) = 0.0050
Ha: mean > 0                                              Pr(T > t) = 0.0025
**Direction of Comparison is Arbitrary**

- Does not impact overall results at all, direction changes, so signs of mean diff and CI endpoints change; but message exactly the same

```
ttesti 14 -.36 .40 0
```

**One-sample t test**

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<tr>
<td>x</td>
<td>14</td>
<td>-.36</td>
<td>.1069045</td>
<td>.4</td>
<td>-.5909531 -.1290469</td>
</tr>
</tbody>
</table>

mean = mean(x)
Ho: mean = 0
t = -3.3675
degrees of freedom = 13

Ha: mean < 0
Pr(T < t) = 0.0025
Ha: mean != 0
Pr(|T| > |t|) = 0.0050
Ha: mean > 0
Pr(T > t) = 0.9975
Summary: Paired t-Test

- Designate null and alternative hypotheses

- Collect data

- Compute difference in outcome for each paired set of observations
  - Compute $\bar{x}_{diff}$, sample mean of the paired differences
  - Compute $s$, sample standard deviation of the differences
Compute 95% (or other level) CI for true mean difference between paired groups compared

- “Big n” \((n > 60)\)

\[
\bar{x}_{dif f} \pm 2 \times \frac{S_{dif f}}{\sqrt{n}}
\]

- “Small n” \((n \leq 60)\)

\[
\bar{x}_{dif f} \pm t_{.95, n-1} \times \frac{S_{dif f}}{\sqrt{n}}
\]
Summary: Paired t-Test

- To get p-values
  - Start by assuming $H_0$ true
  - Measure distance of sample result from $\mu_0$

$$t = \frac{\bar{x}_{diff} - \mu_0}{SE(\bar{x}_{diff})}$$

- Usually, $\mu_0=0$, so:

$$t = \frac{\bar{x}_{diff}}{SE(\bar{x}_{diff})} = \frac{\bar{x}_{diff}}{s_{diff}/\sqrt{n}}$$
Summary: Paired t-Test

- Compare test statistics (distance) to appropriate distribution to get p-value
  - Reminder: p-value measures how likely your sample result (and other result less likely) are if null is true
Summary: Paired t-Test/Paired Data Situations

- Example 1
  - The blood pressure/OC example

- Example 2
  - Degree of clinical agreement, each patient received two assessments

- Example 3
  - Single group of men given two different diets at in two different time periods
  - LDL cholesterol levels measured at end of each diet
Summary: Paired t-Test/Paired Data Situations

- Twin study

- Matched case control scenario
  - Suppose we wish to compare levels of a certain biomarker in patients with a given disease versus those without