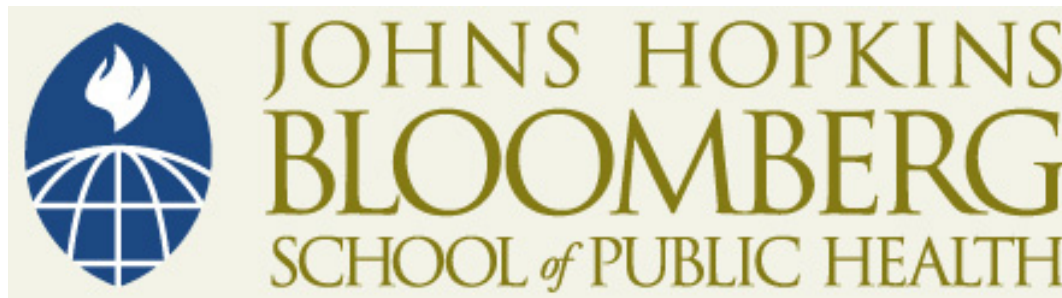


This work is licensed under a [Creative Commons Attribution-NonCommercial-ShareAlike License](https://creativecommons.org/licenses/by-nc-sa/4.0/). Your use of this material constitutes acceptance of that license and the conditions of use of materials on this site.



Copyright 2008, The Johns Hopkins University and Brian Caffo. All rights reserved. Use of these materials permitted only in accordance with license rights granted. Materials provided "AS IS"; no representations or warranties provided. User assumes all responsibility for use, and all liability related thereto, and must independently review all materials for accuracy and efficacy. May contain materials owned by others. User is responsible for obtaining permissions for use from third parties as needed.

Lecture 19

Brian Caffo

Department of Biostatistics
Johns Hopkins Bloomberg School of Public Health
Johns Hopkins University

December 19, 2007

Table of contents

- 1 Table of contents
- 2 Outline
- 3 Relative measures
- 4 The relative risk
- 5 The odds ratio

- 1 Define relative risk
- 2 Odds ratio
- 3 Confidence intervals

Motivation

- Consider a randomized trial where 40 subjects were randomized (20 each) to two drugs with the same active ingredient but different expedients
- Consider counting the number of subjects with side effects for each drug

	Side		
	Effects	None	total
Drug A	11	9	20
Drug B	5	15	20
Total	16	14	40

Comparing two binomials

- Let $X \sim \text{Binomial}(n_1, p_1)$ and $\hat{p}_1 = X/n_1$
- Let $Y \sim \text{Binomial}(n_2, p_2)$ and $\hat{p}_2 = Y/n_2$
- We also use the following notation:

$n_{11} = X$	$n_{12} = n_1 - X$	$n_1 = n_{1+}$
$n_{21} = Y$	$n_{22} = n_2 - Y$	$n_2 = n_{2+}$
n_{2+}	n_{+2}	

- Last time, we considered the absolute change in the proportions, what about relative changes?
- Relative changes are often of more interest than absolute, eg when both proportions are small
- The **relative risk** is defined as p_1/p_2
- The natural estimator for the relative risk is

$$\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{X/n_1}{Y/n_2}$$

- The standard error for $\log \hat{RR}$ is

$$\hat{SE}_{\log \hat{RR}} = \left(\frac{(1-p_1)}{p_1 n_1} + \frac{(1-p_2)}{p_2 n_2} \right)^{1/2}$$

- Exponentiate the resulting interval to get an interval for the RR

- The **odds ratio** is defined as

$$\frac{\text{Odds of SE Drug A}}{\text{Odds of SE Drug B}} = \frac{p_1/(1-p_1)}{p_2/(1-p_2)} = \frac{p_1(1-p_2)}{p_2(1-p_1)}$$

- The sample odds ratio simply plugs in the estimates for p_1 and p_2 , this works out to have a convenient form

$$\hat{OR} = \frac{\hat{p}_1/(1-\hat{p}_1)}{\hat{p}_2/(1-\hat{p}_2)} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

(cross product ratio)

- The standard error for $\log \hat{OR}$ is

$$\hat{SE}_{\log \hat{OR}} = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$$

- Exponentiate the resulting interval to obtain an interval for the OR

Some comments

- Notice that the sample and true odds ratios do not change if we transpose the rows and the columns
- For both the OR and the RR, taking the logs helps with adherence to the error rate
- Of course the interval for the log RR or log OR is obtained by taking

$$\text{Estimate} \pm Z_{1-\alpha/2} SE_{\text{Estimate}}$$

- Exponentiating yields an interval for the OR or RR
- Though logging helps, these intervals still don't perform altogether that well

Example - RR

- For the relative risk, $\hat{p}_A = 11/20 = .55$, $\hat{p}_B = 5/20 = .25$
- $\hat{RR}_{A/B} = .55/.25 = 2.2$
- $\hat{SE}_{\log \hat{RR}_{A/B}} = \sqrt{\frac{1-.55}{.55 \times 20} + \frac{1-.25}{.25 \times 20}} = .44$
- Interval for the log RR:
 $\log(2.2) \pm 1.96 \times .44 = [-.07, 1.65]$
- Interval for the RR: $[.93, 5.21]$

Example - OR

- $\hat{OR}_{A/B} = \frac{11 \times 15}{9 \times 5} = 3.67$
- $\hat{SE}_{\log \hat{OR}_{A/B}} = \sqrt{\frac{1}{11} + \frac{1}{9} + \frac{1}{5} + \frac{1}{15}} = .68$
- Interval for log OR: $\log(3.67) \pm 1.96 \times .68 = [-.04, 2.64]$
- Interval for the OR: $[.96, 14.01]$

Example - RD

- For the risk difference

$$\hat{RD}_{A-B} = \hat{p}_A - \hat{p}_B = .55 - .25 = .30$$

- $\hat{SE}_{\hat{RD}_{A-B}} = \sqrt{\frac{.55 \times .45}{20} + \frac{.25 \times .75}{20}} = .15$

- Interval: $.30 \pm 1.96 \times .15 = [.15, .45]$

