Lecture 1

1. Cover syllabus
   a. mathematical prerequisites
   b. web site
   c. quiz and homework schedule
   d. test schedule
   e. R

2. Abstract the idea of an experiment

3. Develop basic set theory to be used in the development of probability

4. Start discussing probability
Experiments
Consider the outcome of an experiment such as:

• a collection of measurements from a sampled population
• measurements from a laboratory experiment
• the result of a clinical trial
• the result from a simulated (computer) experiment
• values from hospital records sampled retrospectively
• ...
Notation

• The **sample space**, \( \Omega \), is the collection of possible outcomes of an experiment
  
  Example: die roll \( \Omega = \{1, 2, 3, 4, 5, 6\} \)

• An **event**, say \( E \), is a subset of \( \Omega \)
  
  Example: die roll is even \( E = \{2, 4, 6\} \)

• An **elementary** or **simple** event is a particular result of an experiment
  
  Example: die roll is a four, \( \omega = 4 \)

• \( \emptyset \) is called the **null event** or the **empty set**
Interpretation of set operations
Normal set operations have particular interpretations in this setting

1. \( \omega \in E \) implies that \( E \) occurs when \( \omega \) occurs
2. \( \omega \notin E \) implies that \( E \) does not occur when \( \omega \) occurs
3. \( E \subset F \) implies that the occurrence of \( E \) implies the occurrence of \( F \)
4. \( E \cap F \) implies the event that both \( E \) and \( F \) occur
5. \( E \cup F \) implies the event that at least one of \( E \) or \( F \) occur
6. \( E \cap F = \emptyset \) means that \( E \) and \( F \) are mutually exclusive, or cannot both occur
7. \( E^c \) or \( \bar{E} \) is the event that \( E \) does not occur
**Fun aside: Russell’s paradox**

Russell’s paradox is one of the most famous results of set theory

- Consider, $R$, the set containing all sets that do not contain themselves as an element

  Alternatively, consider writing down a catalog of all catalogs who do not have themselves listed as an entry

- Does $R$ contain itself?
  
  → If yes, then $R$ is not allowed to be in $R$, by the definition
  
  → If no, then $R$ has to be in $R$, by the definition
Set theory facts

• DeMorgan’s laws

\[(A \cap B)^c = A^c \cup B^c\]
\[(A \cup B)^c = A^c \cap B^c\]

Example: If an alligator or a turtle you are not\[ (A \cup B)^c \] then you are not an alligator and you are also not a turtle \[ (A^c \cap B^c) \]

Example: If your car is not both hybrid and diesel \[ (A \cap B)^c \] then your car is either not hybrid or not diesel \[ (A^c \cup B^c) \]

• \[ (A^c)^c = A \]

• \[ (A \cup B) \cap C = (A \cap C') \cup (A \cap B) \]
Probability: some discussion

• Useful strategy used in much of science:
  For a given experiment
  ► attribute all that is known or theorized to a mecha-
    nistic model (mathematical function)
  ► attribute everything else to randomness, even if the
    process under study is known not to be “random” in
    any sense of the word
  ► Use probability to quantify the uncertainty in your
    conclusions
  ► Evaluate the sensitivity of your conclusions to the
    assumptions of your model
Probability: some discussion

- Probability has been found extraordinarily useful, even if true *randomness* is an elusive, undefined, quantity.
- *Frequentist* interpretation of probability
  - A probability is the long proportion of times an event will occur in repeated identical repetitions of an experiment.
- Other definitions of probability exist.
- There is not agreement, at all, in how probabilities should be interpreted.
- There is (nearly) complete agreement on the mathematical rules probability must follow.
Probability: some discussion

* An alternative interpretation of probability is so-called “Bayesian”
* Named after the 18th century Presbyterian Minister / mathematician Thomas Bayes
* Bayesian interprets probability as a subjective degree of belief
  * For the same event, two separate people could have differing probabilities
  * Bayesian interpretations of probabilities avoid some of the philosophical difficulties of frequency interpretations