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Section C

The Paired t-Test; Two More Examples

Clinical Agreement by Two Diagnosing Physicians

- Two different physicians assessed the number of palpable lymph nodes in 65 randomly selected male sexual contacts of men with AIDS or AIDS-related conditions[1]

	Doctor 1	Doctor 2	Difference
Mean ($\bar{x}$)	7.91	5.16	–2.75
sd (s)	4.35	3.93	2.83

[1]Example based on data taken from Rosner, B. (2005). *Fundamentals of Biostatistics*, sixth. ed. Duxbury Press. (Based on research by Coates, et al. (1988). Assessment of generalized … *Journal of Clinical Epidemiology*, 41(2).

95% Confidence Interval

- 95% CI for difference in mean number of lymph nodes, Doctor 2 compared to Doctor 1

$$\bar{x}_{diff} \pm 2 \times S\hat{E}(\bar{x}_{diff})$$

$$\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{65}}$$

$$2.75 \pm 2 \times \left(\frac{2.83}{\sqrt{65}} \right)$$

$$-3.45 \;\; to \;\; -2.05$$

Getting a p-Value

- Hypotheses
  - H_o: $\mu_{diff} = 0$
  - H_A: $\mu_{diff} \neq 0$

- First, start by "assuming" null is true and computing distance (in SEs) between $\bar{x}_{diff}$ and 0
  - Sample result is 7.8 SEs below 0—*is this unusual?*

$$t = \frac{\bar{x}_{diff} - 0}{\hat{SE}(\bar{x})} = \frac{-2.75}{2.83/\sqrt{65}} = -7.8$$

Getting a p-Value

- Sample result is 7.8 SEs below 0—*is this unusual?*
  - See where this falls on sampling distribution of all possible mean differences based on random samples of 65 patients
    - Theory tells us this is normal

- The p-value is probability of being 7.8 or more standard errors from 0 under a standard normal curve
  - Without looking up, we know p <<< .001!

Everything with Stata

- ttesti 65 -2.75 2.83 0

```
. ttesti  65 -2.75 2.83 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      65       -2.75    .3510183        2.83   -3.45124    -2.04876
------------------------------------------------------------------------------
    mean = mean(x)                                                t =  -7.8343
Ho: mean = 0                                     degrees of freedom =       64

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 1.0000
```

Oat Bran and LDL Cholesterol

- Cereal and cholesterol: 14 males with high cholesterol given oat bran cereal as part of diet for two weeks, and corn flakes cereal as part of diet for two weeks

	Corn Flakes	**Oat Bran**	**Difference**
Mean ($\bar{x}$)	4.44 mmol/dL	4.08	0.36
sd (s)	1.0	1.1	0.40

[1]Example based on data taken from Pagano, M. (2000). *Principles of Biostatistics*, 2nd ed. Duxbury Press. Based on research by Anderson J, et al. (1990). Oat Bran Cereal Lowers … *American Journal of Clinical Nutrition*, 52.

95% Confidence Interval

- 95% CI for difference in mean LDL, corn flakes vs. oat bran

$$\bar{x}_{diff} \pm t_{.95,13} \times \hat{SE}(\bar{x}_{diff})$$

$$\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{14}}$$

$$0.36 \pm 2 \times \left(\frac{.040}{\sqrt{14}} \right)$$

$$0.13 \;\; to \;\; 0.60 \; mmol/dL$$

Getting a p-Value

- Hypotheses
  - H_o: $\mu_{diff} = 0$
  - H_A: $\mu_{diff} \neq 0$

- First, start by "assuming" null is true, and computing distance (in SEs) between $\bar{x}_{diff}$ and 0
  - Sample result is 3.3 SEs above 0—*is this unusual?*

$$t = \frac{\bar{x}_{diff} - 0}{\hat{SE}(\bar{x})} = \frac{.036}{.04/\sqrt{14}} \approx 3.3$$

Getting a p-Value

- Sample result is 3.3 SEs above 0—*is this unusual?*
  - See where this falls on sampling distribution of all possible mean differences based on random samples of 14 patients: theory tells us this is t_{13}

- The p-value is probability of being 3.3 or more standard errors from 0 under a t_{13} curve: look up in table or go to Stata

Everything with Stata

- cii 14 .36 .40 0

```
. ttesti 14 .36 .40 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      14         .36    .1069045          .4    .1290469    .5909531
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   3.3675
Ho: mean = 0                                     degrees of freedom =       13

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.9975         Pr(|T| > |t|) = 0.0050          Pr(T > t) = 0.0025
```

Direction of Comparison is Arbitrary

- Does not impact overall results at all, direction changes, so signs of mean diff and CI endpoints change; but message exactly the same

```
. ttesti 14 -.36 .40 0

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      14        -.36    .1069045          .4   -.5909531   -.1290469
------------------------------------------------------------------------------
    mean = mean(x)                                                t =  -3.3675
Ho: mean = 0                                     degrees of freedom =       13

    Ha: mean < 0                 Ha: mean != 0                 Ha: mean > 0
 Pr(T < t) = 0.0025         Pr(|T| > |t|) = 0.0050          Pr(T > t) = 0.9975
```

Summary: Paired t-Test

- Designate null and alternative hypotheses

- Collect data

- Compute difference in outcome for each paired set of observations
  - Compute $\bar{x}_{diff}$, sample mean of the paired differences
  - Compute *s*, sample standard deviation of the differences

Summary: Paired t-Test

- Compute 95% (or other level) CI for true mean difference between paired groups compared
  - "Big n" ($n > 60$)

$$\bar{x}_{diff} \pm 2 \times \frac{s_{diff}}{\sqrt{n}}$$

  - "Small n" ($n \leq 60$)

$$\bar{x}_{diff} \pm t_{.95,n-1} \times \frac{s_{diff}}{\sqrt{n}}$$

Summary: Paired t-Test

- To get p-values
  - Start by assuming H_o true
  - Measure distance of sample result from μ_o

$$t = \frac{\bar{x}_{diff} - \mu_o}{\hat{SE}(\bar{x}_{diff})}$$

  - Usually, μo=0, so:

$$t = \frac{\bar{x}_{diff}}{\hat{SE}(\bar{x}_{diff})} = \frac{\bar{x}_{diff}}{s_{diff}/\sqrt{n}}$$

Summary: Paired t-Test

- Compare test statistics (distance) to appropriate distribution to get p-value
  - Reminder: p-value measures how likely your sample result (and other result less likely) are if null is true

Summary: Paired t-Test/Paired Data Situations

- Example 1
  - The blood pressure/OC example
- Example 2
  - Degree of clinical agreement, each patient received two assessments
- Example 3
  - Single group of men given two different diets at in two different time periods
  - LDL cholesterol levels measured at end of each diet

Summary: Paired t-Test/Paired Data Situations

- Twin study

- Matched case control scenario
  - Suppose we wish to compare levels of a certain biomarker in patients with a given disease versus those without