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Comparing Means among Two (or More) Independent Populations

John McGready
Johns Hopkins University

Lecture Topics

- CIs for mean difference between two independent populations

- Two sample t-test

- Non-parametric alternative, Mann Whitney (FYI, optional)

- Comparing means amongst more than two independent populations: ANOVA

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Section A

Two Sample t-test: The Resulting Confidence Interval

Comparing Two Independent Groups

- "A Low Carbohydrate as Compared with a Low Fat Diet in Severe Obesity"*
  - 132 severely obese subjects randomized to one of two diet groups
  - Subjects followed for a six month period

- At the end of study period
  - "Subjects on the low-carbohydrate diet lost more weight than those on a low fat diet (95% confidence interval for the difference in weight loss between groups, -1.6 to -6.2 kg; $p < .01$)"

Source: * Samaha, F., et al. A low-carbohydrate as compared with a low-fat diet in severe obesity, *New England Journal of Medicine*, 348: 21.

Comparing Two Independent Groups: Diet Types Study

- Scientific question
  - Is weight change associated with diet type?

	Diet Group	
	Low-Carb	Low-Fat
Number of subjects (n)	64	68
Mean weight change (kg) Post-diet less pre-diet	-5.7	-1.8
Standard deviation of weight changes (kg)	8.6	3.9

Diet Type and Weight Change

- 95% CIs for weight change by diet group

- Low Carb: $-5.7 \pm 2 \times \frac{8.6}{\sqrt{64}} \rightarrow -5.7 \pm 2 \times 1.08 \approx (-7.8\,kg, -3.5\,kg)$

- Low Fat: $-1.8 \pm 2 \times \frac{3.9}{\sqrt{68}} \rightarrow -1.8 \pm 2 \times .47 \approx (-2.7\,kg, -0.9\,kg)$

Comparing Two Independent Groups: Diet Types Study

- In statistical terms, is there a non-zero difference in the average weight change for the subjects on the low-fat diet as compared to subjects on the low-carbohydrate diet?
  - 95% CIs for each diet group mean weight change do not overlap, but how do you quantify for the difference?

- The comparison of interest is not "paired"
  - There are different subjects in each diet group

- For each subject a change in weight (after diet—before weight) was computed
  - However, the authors compared the changes in weight between two independent groups!

Comparing Two Independent Groups

- How do we calculate
  - Confidence interval for difference?
  - p-value to determine if the difference in two groups is "significant?"

- Since we have large samples (both greater than 60) we know the sampling distributions of the sample means in both groups are approximately normal

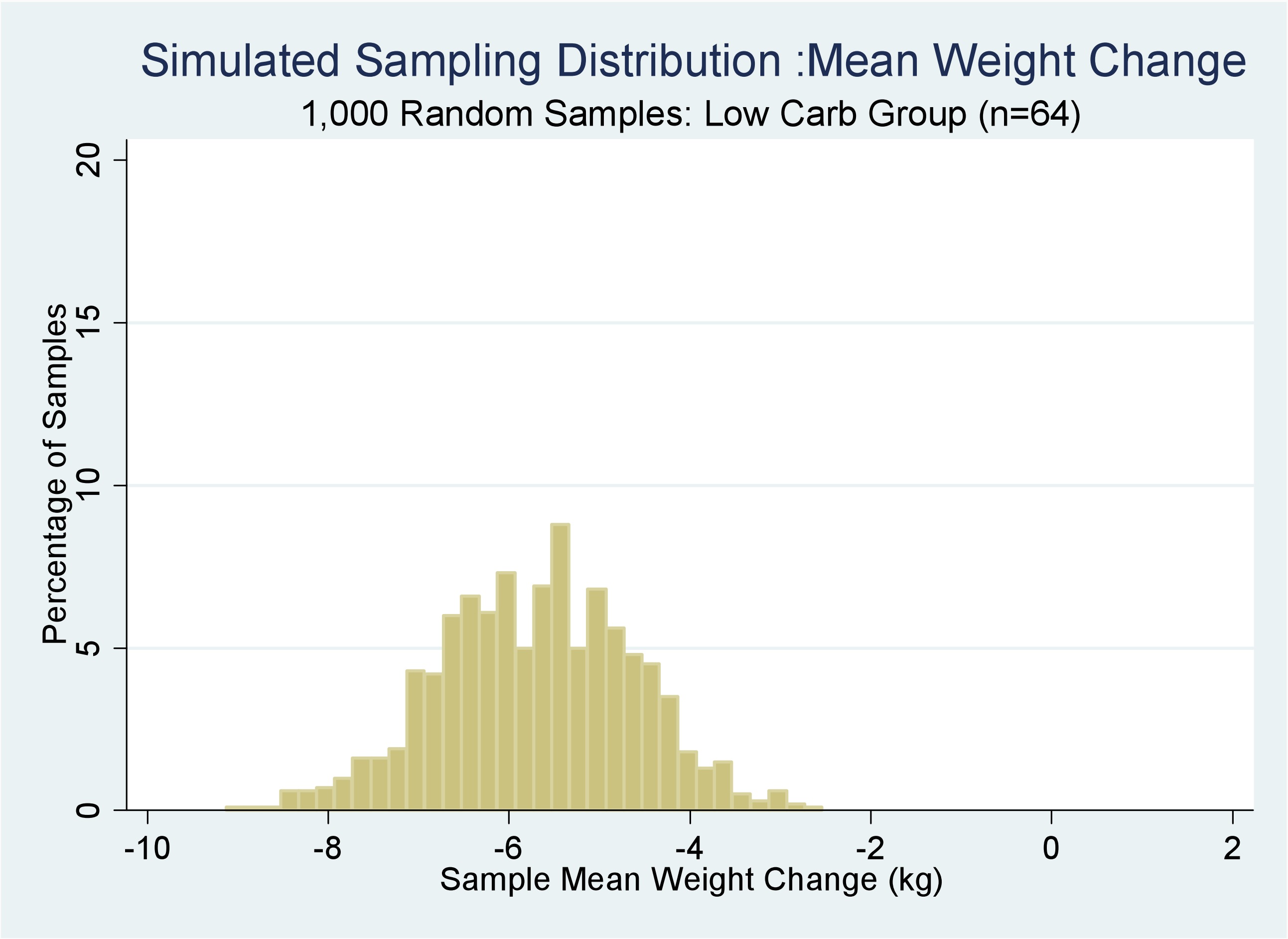
- It turns out the difference of quantities, which are (approximately) normally distributed, are also normally distributed

Sampling Distribution: Difference in Sample Means

- So, the big news is . . .
  - The sampling distribution of the difference of two sample means, each based on large samples, approximates a normal distribution
  - This sampling distribution is centered at the true mean difference, $\mu_1 - \mu_2$

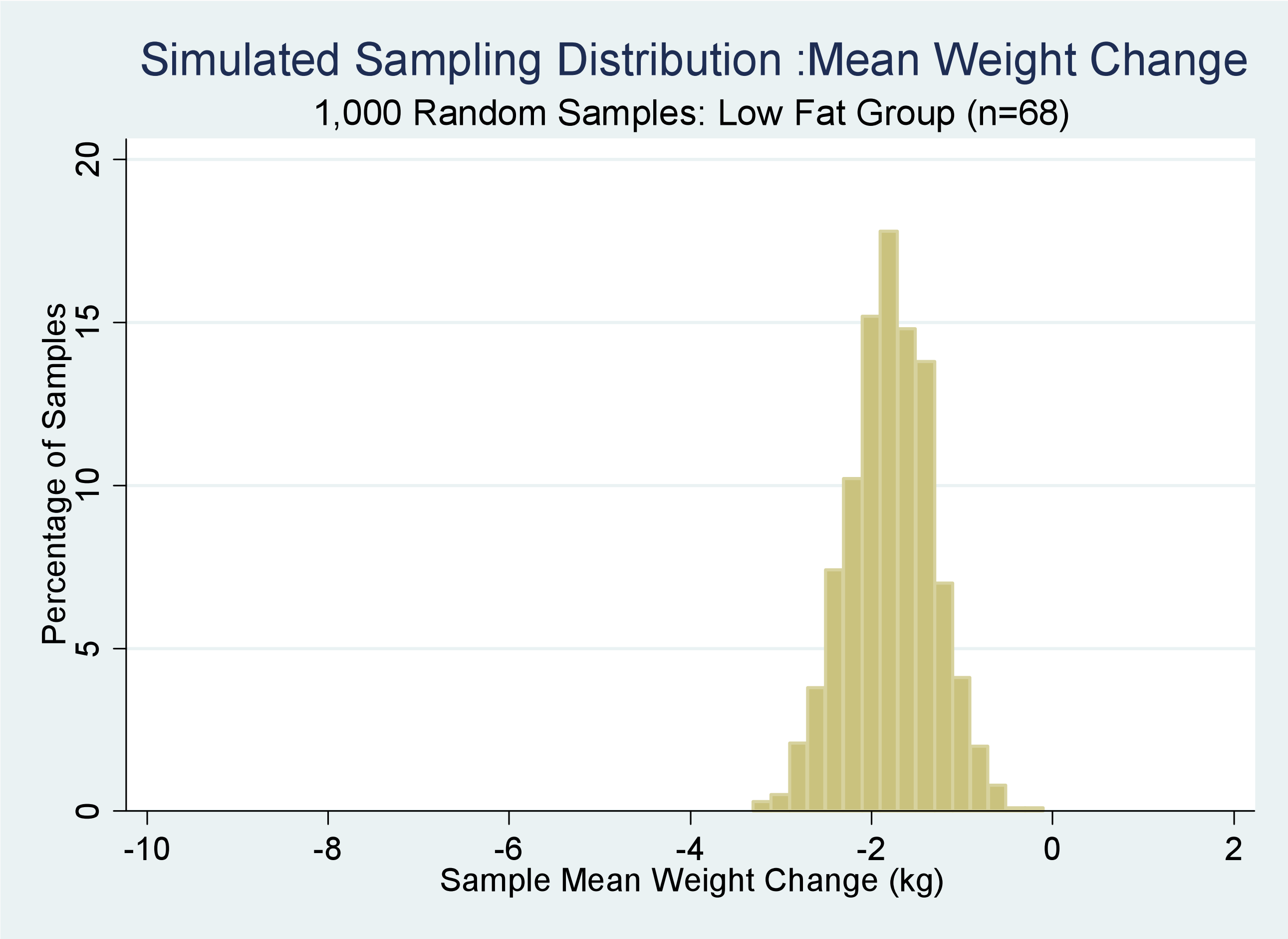
Simulated Sampling Dist'n of Sample Mean Weight Loss

- Simulated sampling distribution of sample mean weight change: low carbohydrate diet group



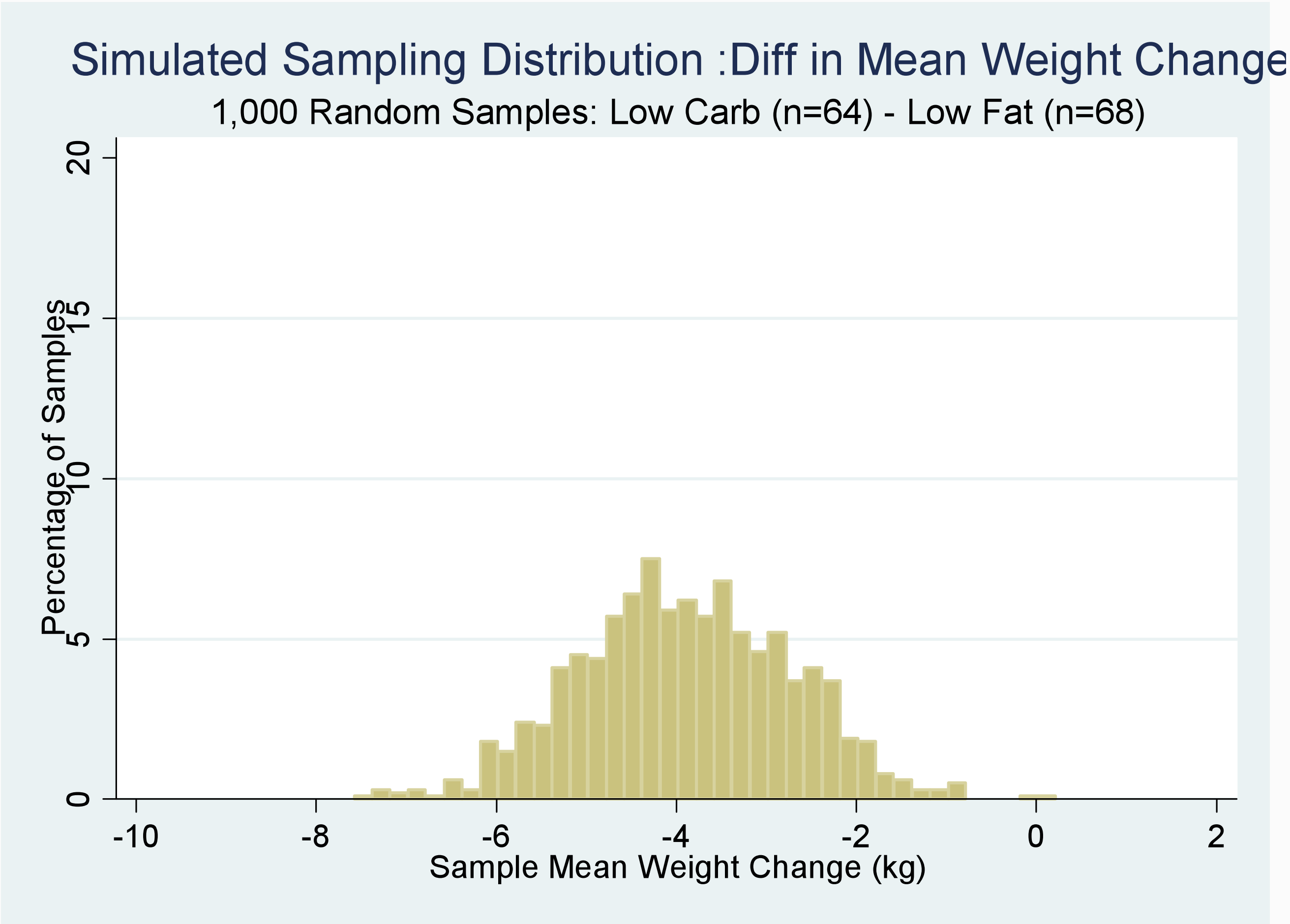
Simulated Sampling Dist'n of Sample Mean Weight Loss

- Simulated sampling distribution of sample mean weight change: low fat diet group



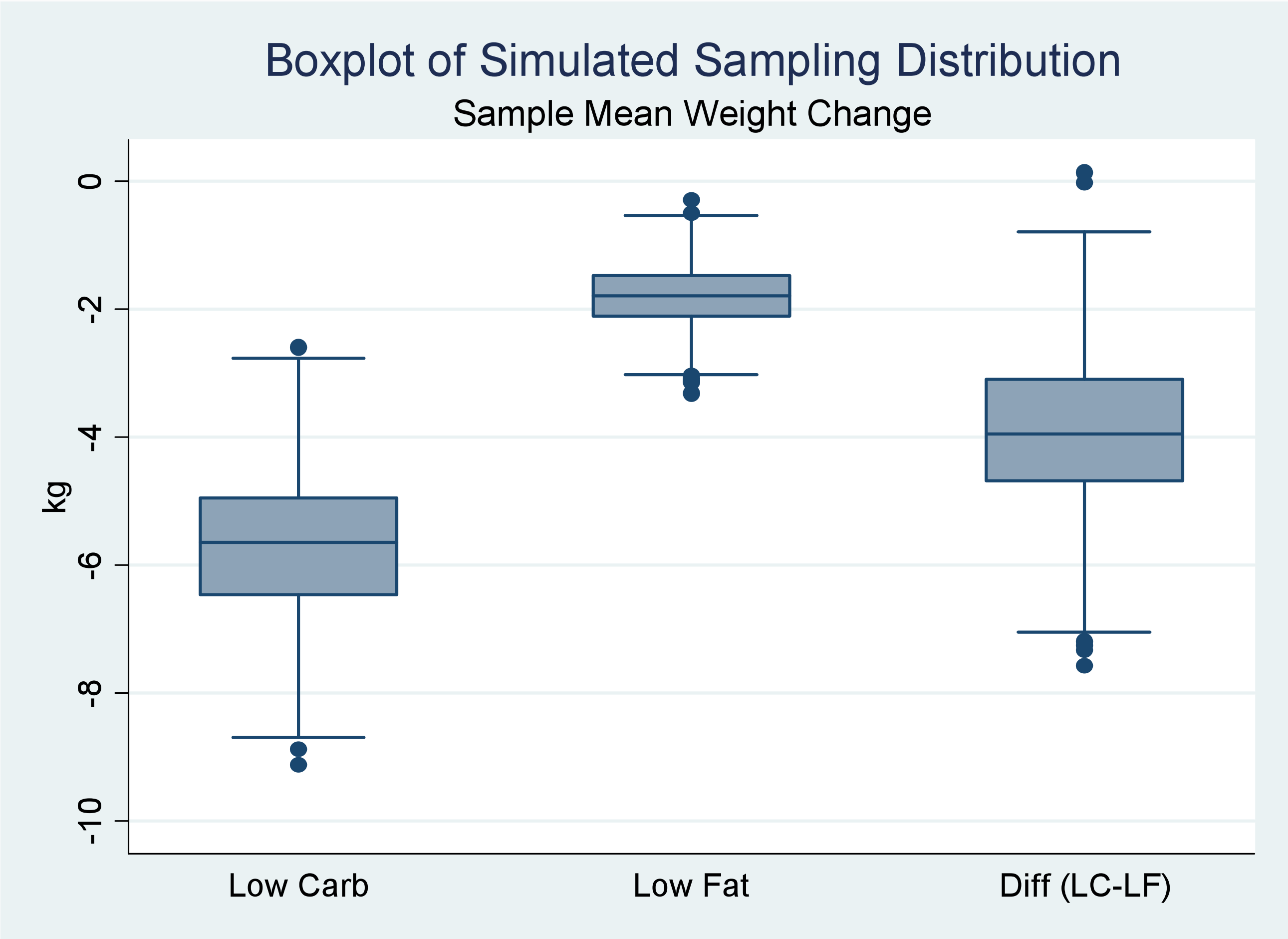
Simulated Sampling Dist'n of Sample Mean Weight Loss

- Simulated sampling distribution of sample mean weight change: low fat diet group



Simulated Sampling Dist'n of Sample Mean Weight Loss

- Side by side boxplots



95% Confidence Interval for Difference in Means

- Our most general formula

$$best\ estimate\ from\ sample \pm 2 \times SE(best\ estimate\ from\ sample)$$

- The best estimate of a population mean difference based on sample means:

$$\bar{x}_1 - \bar{x}_2$$

- Here, $\bar{x}_1$ may represent the sample mean weight loss for the 64 subjects on the low carbohydrate diet, and $\bar{x}_2$ the mean weight less for the 68 subjects on the low fat diet

95% CI for Difference in Means: Diet Types Study

- So, $\bar{x}_1 - \bar{x}_2 = -5.7 - (-1.8) = -3.9\ kg$: hence the formula for the 95% CI for μ_1 - μ_2 is:

$$-3.9 \pm 2 \times SE(\bar{x}_1 - \bar{x}_2)$$

- Where $SE(\bar{x}_1 - \bar{x}_2)$ = standard error of the difference of two sample means

Two Independent (Unpaired) Groups

- The standard error of the difference for two independent samples is calculated differently than we did for paired designs
  - With paired design we reduced data on two samples to one set of differences between two groups

- Statisticians have developed formulas for the standard error of the difference

- These formulas depend on sample sizes in both groups and standard deviations in both groups

- The $SE(\bar{x}_1 - \bar{x}_2)$ is greater than either $SE(\bar{x}_1)$ or $SE(\bar{x}_2)$
  - Why do you think this is?

Principle

- Variation from independent sources can be added
  - Why do you think this is additive

$$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^{\;2}}{n_1} + \frac{\sigma_2^{\;2}}{n_2}}$$

- Of course, we don't know σ_1 and σ_2: so we estimate with s1 and s2 to get an estimated standard error:

$$S\hat{E}(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^{\;2}}{n_1} + \frac{s_2^{\;2}}{n_2}} = \sqrt{S\hat{E}(\bar{x}_1)^2 + S\hat{E}(\bar{x}_2)^2}$$

Comparing Two Independent Groups: Diet Types Study

- Recall the data from the weight change/diet type study

	Diet Group	
	Low-Carb	Low-Fat
Number of subjects (n)	64	68
Mean weight change (kg) Post-diet less pre-diet	-5.7	-1.8
Standard deviation of weight changes (kg)	8.6	3.9

$$\hat{SE}(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{8.6^2}{64} + \frac{3.9^2}{68}} \approx 1.17$$

95% CI for Difference in Means: Diet Types Study

- So in this example, the estimated 95% for the true mean difference in weight between the low-carbohydrate and low-fat diet groups is:

$$-3.9 \pm 2 \times S\hat{E}(\bar{x}_1 - \bar{x}_2)$$

$$-3.9 \pm 2 \times 1.17$$

$$-3.9 \pm 2 \times 1.17$$

$$-6.24\ kg\ to\ \text{-}\ 1.56\ kg \approx$$

$$-6.2\ kg\ to\ \text{-}\ 1.6\ kg$$

From Article

- "Subjects on the low-carbohydrate diet lost more weight than those on a low fat diet (95% confidence interval for the difference in weight loss between groups, -1.6 to -6.2 kg; $p < .01$)"

- So those on the low carb diet lost more on average by 3.9 kg: after accounting for sampling variability this excess average loss over the low-fat diet group could be as small as 1.6 kg or as large as 6.2 kg
  - This confidence interval does not include 0, suggesting a real population level association between type of diet (low-carb or low-fat) and weight loss