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Simple Linear Regression

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Section A

Review: The Equation of a Line

The Equation of a Line

- Recall, from algebra, there are two values which uniquely define any line
  - Y-intercept—where the line crosses the y-axis (when x = 0)
  - Slope—the “rise over the run”—how much y changes for each one unit change in x

The Equation of a Line

- Recall, from algebra, there are two values which uniquely define any line

- $y = mx + b$
  - b = y-intercept
  - m = slope

The Equation of a Line

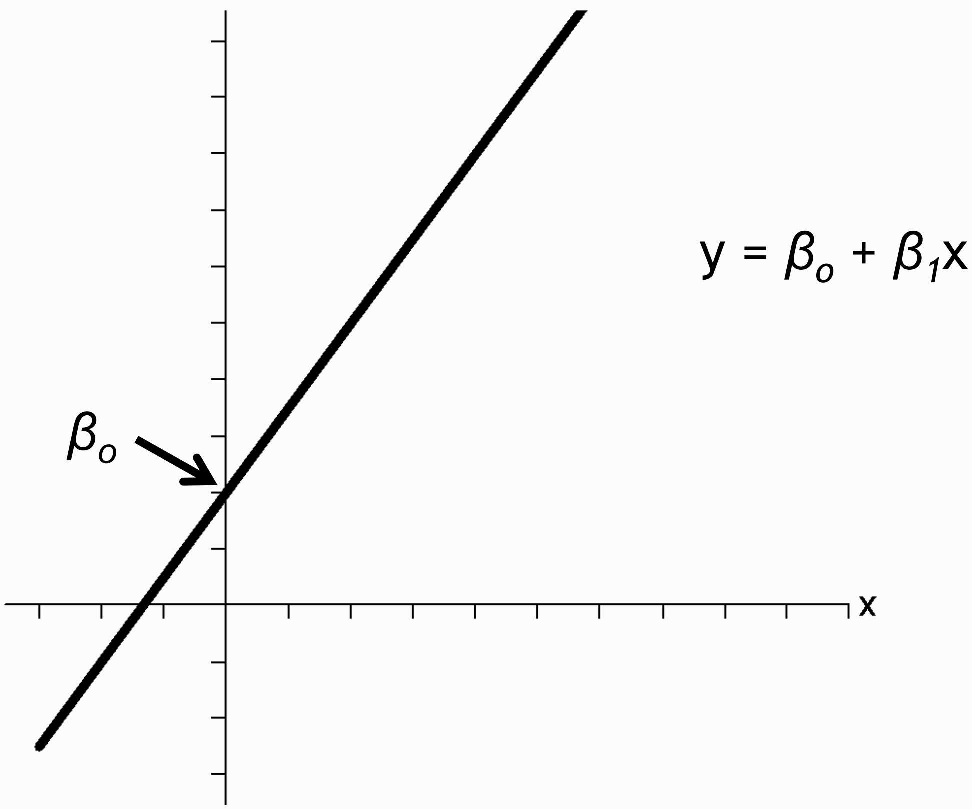
- Of course statisticians must have their own notation!

- $y = b_o + b_1 x$
  - b_o = y-intercept
  - b_1 = slope

- $y = \beta_o + \beta_1 x$
  - β_o = y-intercept
  - β_1 = slope

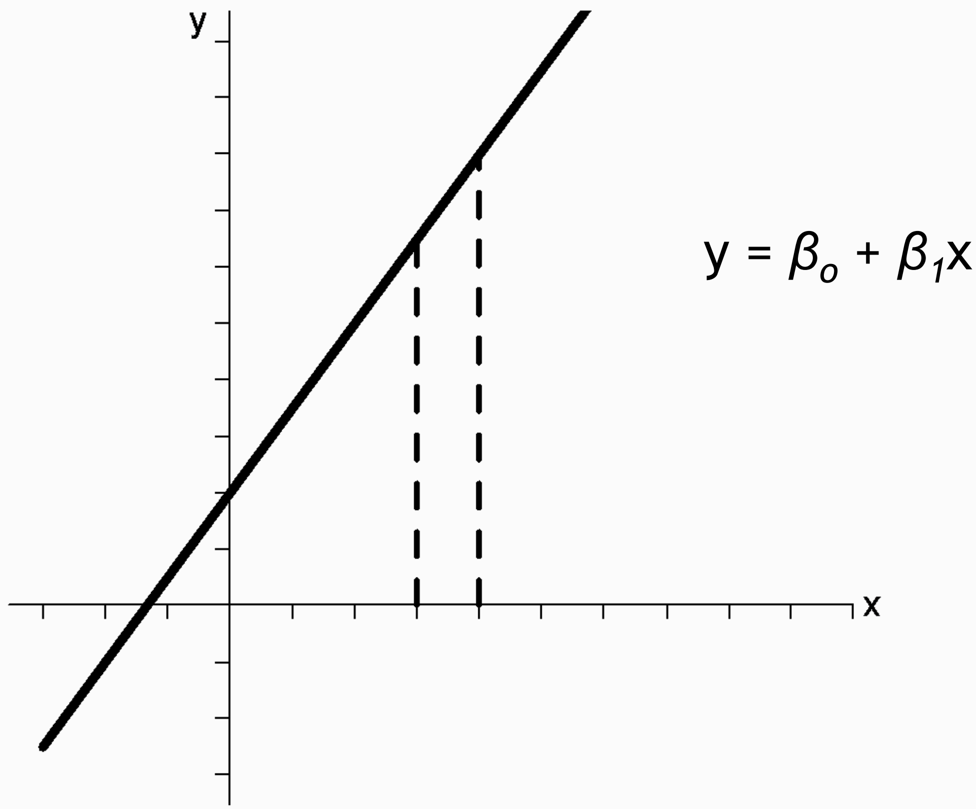
The Intercept, β_o

- The intercept β_o is the value of y when x is 0
  - It is the point on the graph where the line crosses the y (vertical) axis, at the coordinate $(0, \beta_o)$



The Slope, β_1

- The slope β_1 is the change in y corresponding to a unit increase in x



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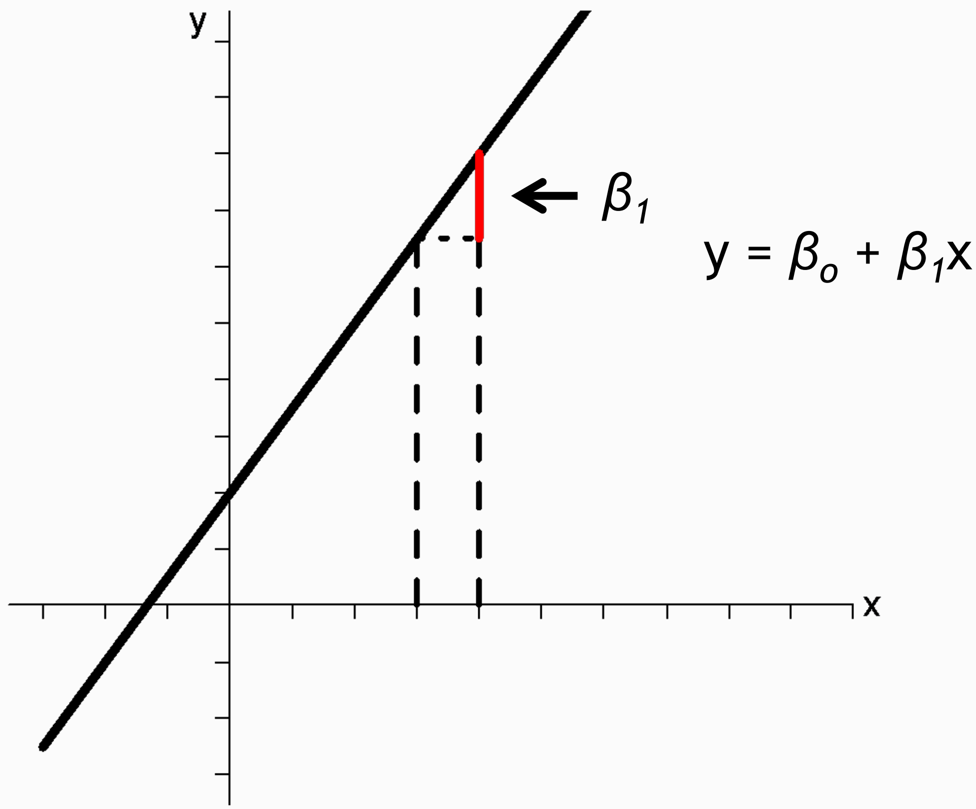
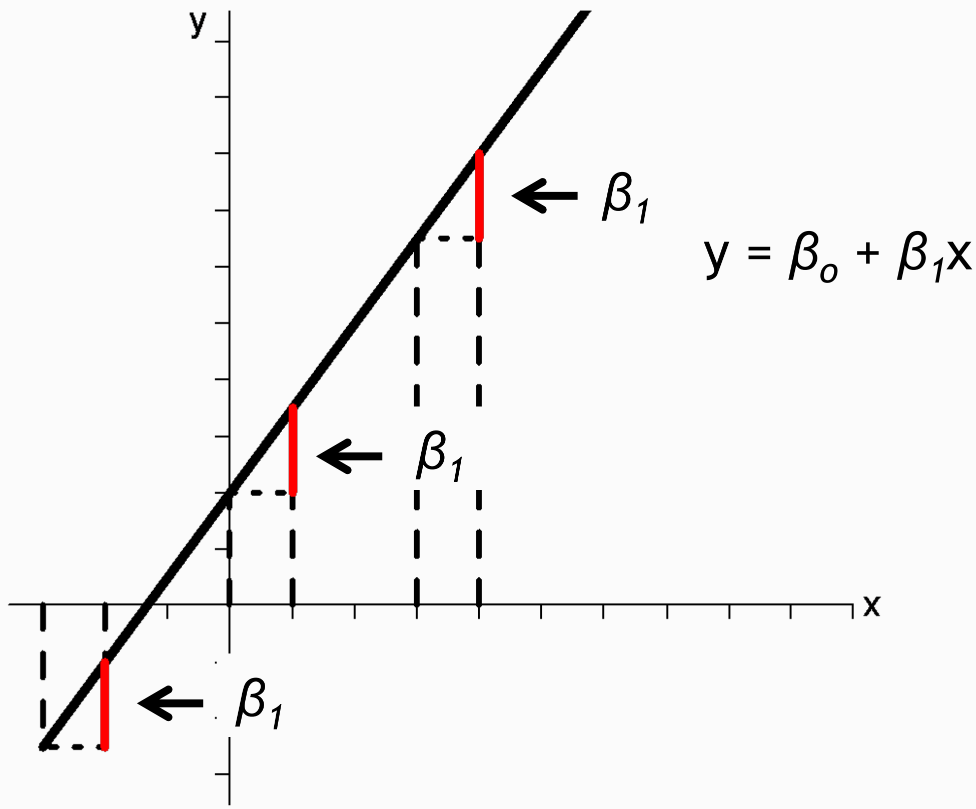


The Slope, β_1

- The slope β_1 is the change in y corresponding to a unit increase in x
- Another interpretation: β_1 is difference in y-values for x+1 compared to x
- This change/difference is the same across the entire line

The Slope, β_1

- The slope β_1 is the change in y corresponding to a unit increase in x



The Slope, β_1

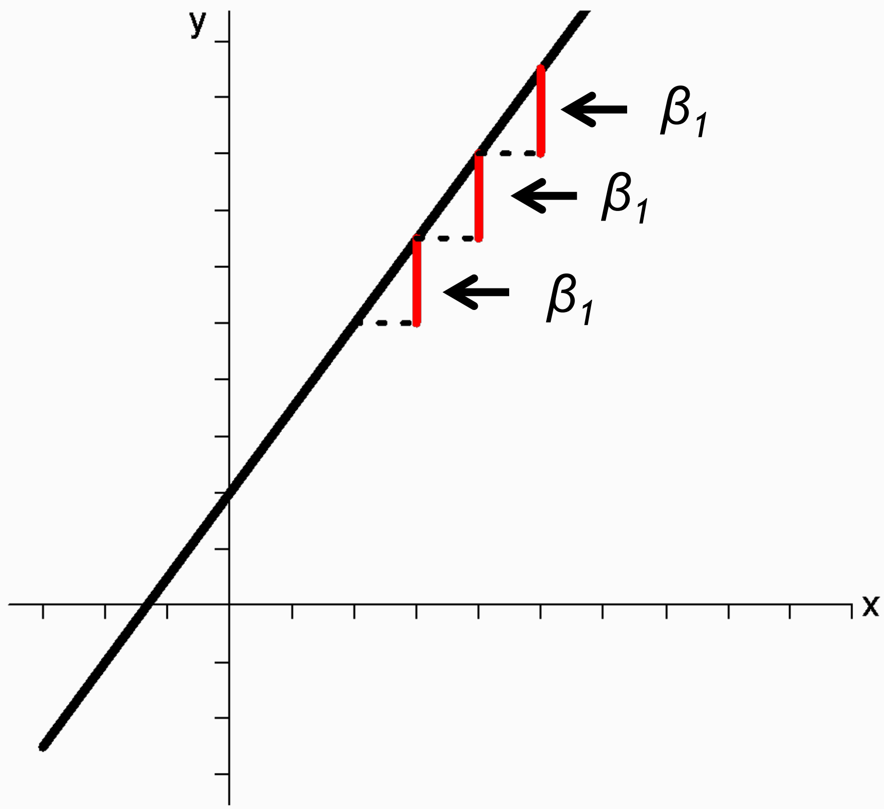
- The slope β_1 is the change in y corresponding to a unit increase in x: β_1 is difference in y-values for x+1 compared to x

- All information about the difference in the y-value for two differing values of x is contained in the slope!

- For example: two values of x three units apart will have a difference in y values of 3* β_1

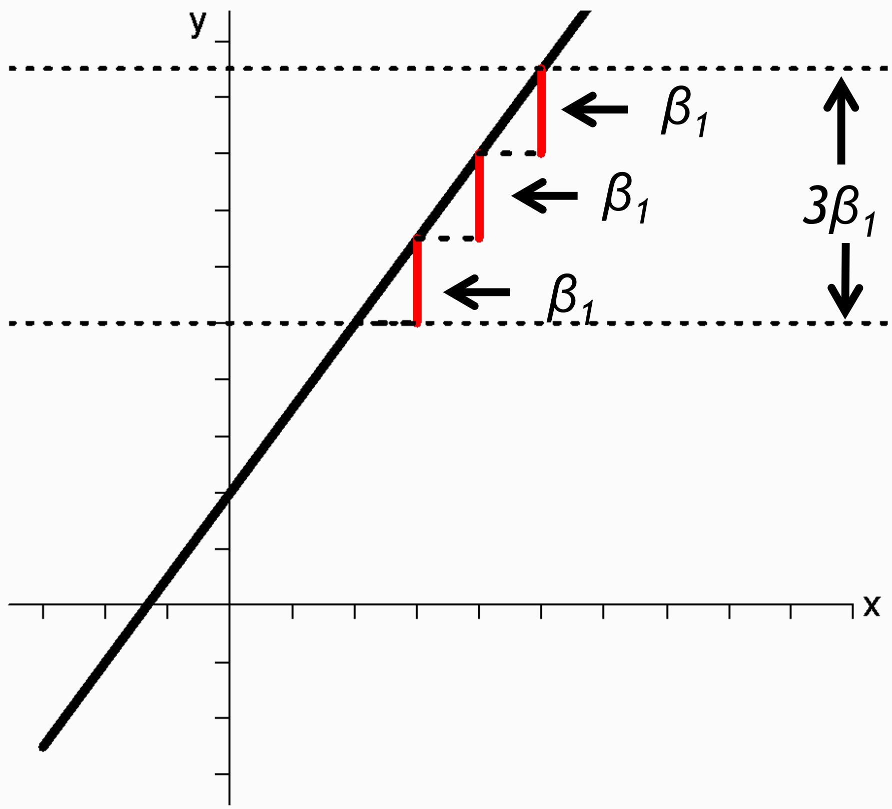
The Slope, β_1

- For example: two values of x three units apart will have a difference in y values of 3* β_1



The Slope, β_1

- For example: two values of x three units apart will have a difference in y values of $3\times\beta_1$ ($3\beta_1$)

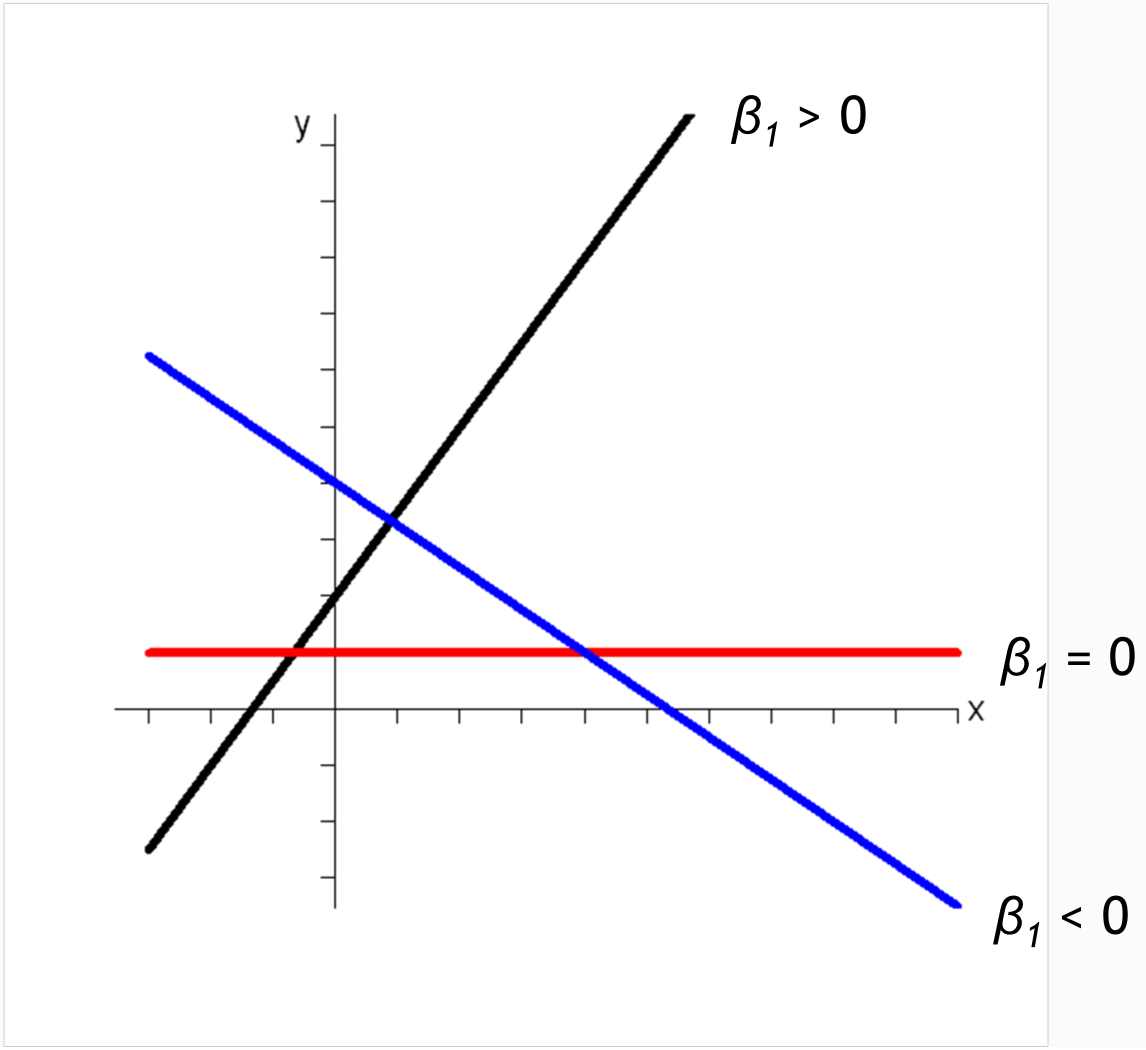


The Slope, β_1

- The slope β_1 is the change in y corresponding to a unit increase in x: β_1 is difference in y-values for x+1 compared to x
  - If slope $\beta_1 = 0$, this indicates that there is no association: (i.e., the values of y are the same regardless of the values of x)
  - If slope $\beta_1 > 0$, this indicates that there is a positive association: (i.e., the values of y increase with increasing values of x)
  - If slope $\beta_1 < 0$, this indicates that there is a negative association: (i.e., the values of y decrease with increasing values of x)

The Slope, β_1

- The slope β_1 is the change in y corresponding to a unit increase in x: β_1 is difference in y-values for x+1 compared to x



The Equation of a Line

- In linear regression situations, points don't fit exactly to a line

- We estimate a line that relates the mean of an outcome y to a predictor x

$$E[y] = \hat{\beta}_0 + \hat{\beta}_1 x$$

  - E[y] = estimated "expected" (mean) value of y
  - $\hat{\beta}_0$ = estimated y-intercept
  - $\hat{\beta}_1$ = estimated slope

The Equation of a Line

- $\hat{\beta}_o$ and $\hat{\beta}_1$ are called estimated regression coefficients

- These two quantities are estimated using the data
  - Line estimated is line that “fits the data best”

- Many times the equation is just written as the following:

$$y = \hat{\beta}_0 + \hat{\beta}_1 x$$

or

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

The Equation of a Line

- $\hat{\beta}_o$ and $\hat{\beta}_1$ are called estimated regression coefficients

- We will see that in a regression context, $\hat{\beta}_1$ is nothing more than estimated mean difference in y between two groups who differ by one unit in x
  - i.e., how much the mean of y changes for a one-unit increase in x