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Section B

Linear Regression: Motivating Example

Example: Arm Circumference and Height

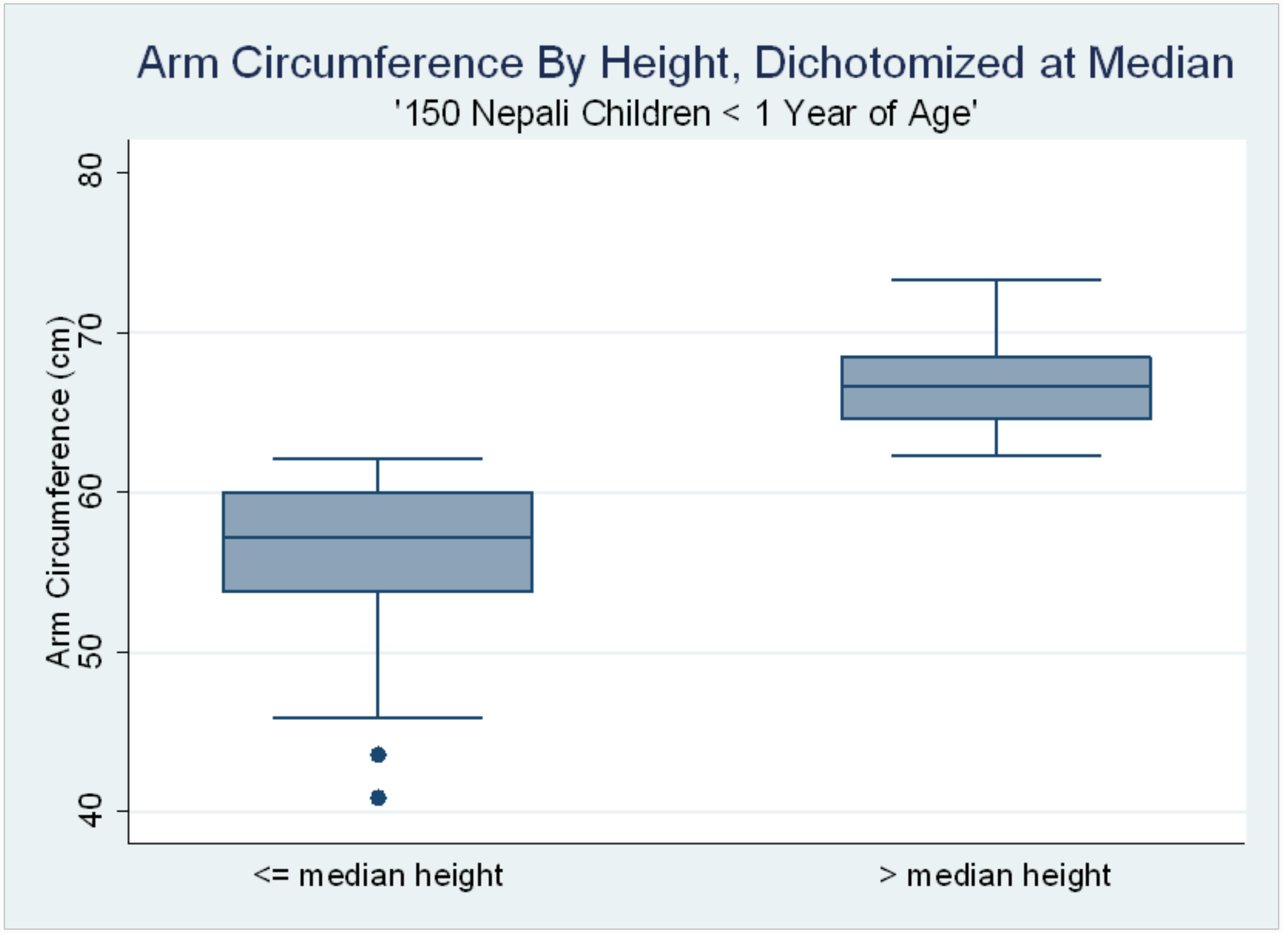
- Data on anthropomorphic measures from a random sample of 150 Nepali children [0, 12) months old

- Question: what is the relationship between average arm circumference and height

- Data:
  - Arm circumference: mean 12.4 cm, SD 1.5 cm, range 7.3 cm – 15.6 cm
  - Height: mean 61.6 cm, SD 6.3 cm, range 40.9 cm – 73.3 cm

Approach 1: Arm Circumference and Height

- Dichotomize height at median, compare mean arm circumference with t-test and 95% CI



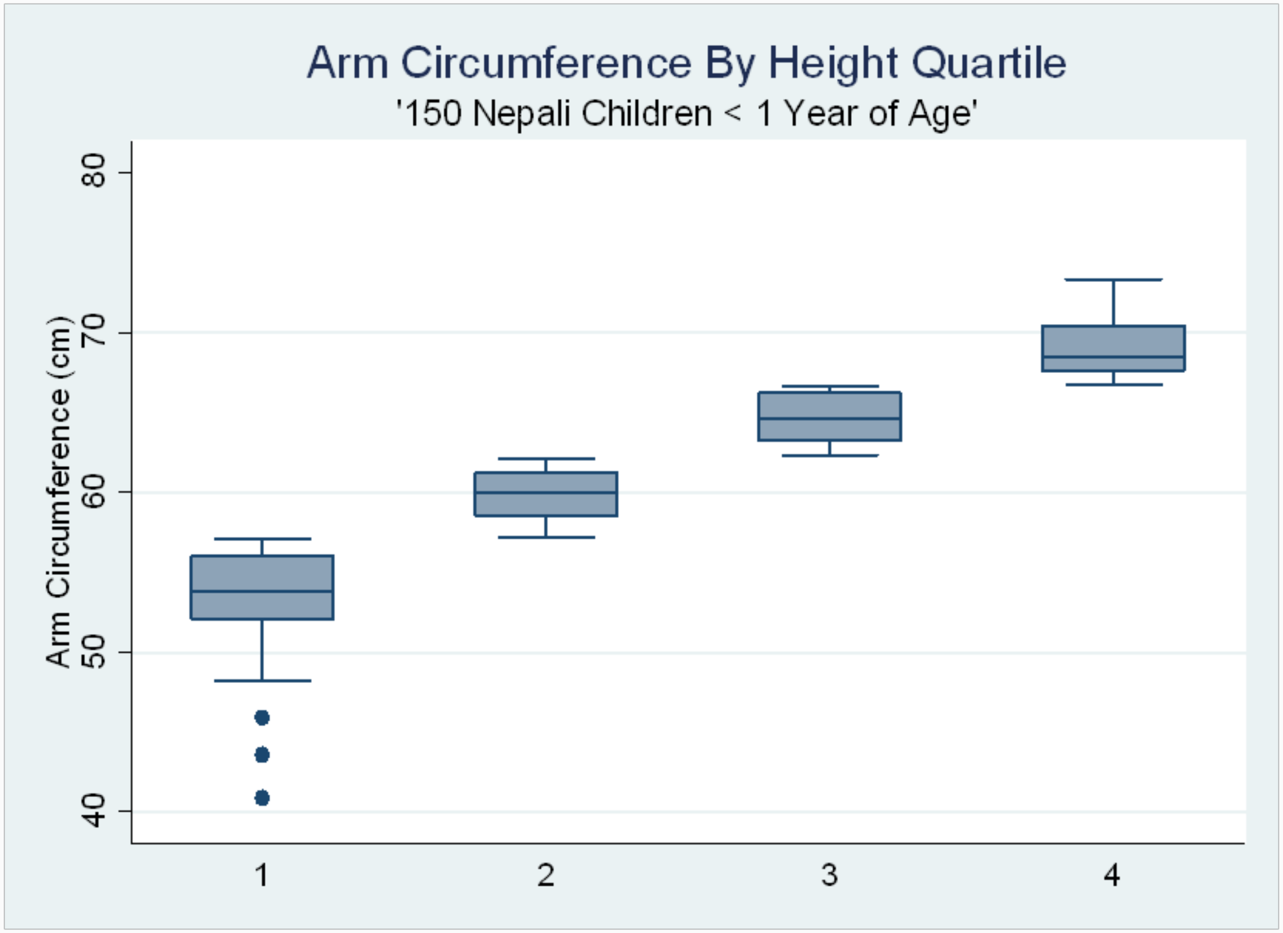
Approach 1: Arm Circumference and Height

- Potential advantages:
  - We know how to do it!
  - Gives a single summary measure (sample mean difference) for quantifying the arm circumference/height association

- Potential disadvantages:
  - Throws away a lot of information in the height data that was originally measured as continuous
  - Only allows for a single comparison between two crudely defined height categories

Approach 2: Arm Circumference and Height

- Categorize height into four categories by quartile, compare mean arm circumference with ANOVA, 95% CIs



Approach 2: Arm Circumference and Height

- Potential advantages:
  - We know how to do it!
  - Uses a less crude categorization of height than the previous approach of dichotomizing

- Potential disadvantages:
  - Still throws away a lot of information in the height data that was originally measured as continuous
  - Requires multiple summary measures (six sample mean differences between each unique combination of height categories) to quantify arm circumference/height relationship
  - Does not exploit the structure we see in the previous boxplot: as height increases so does arm circumference

Approach 3: Arm Circumference and Height

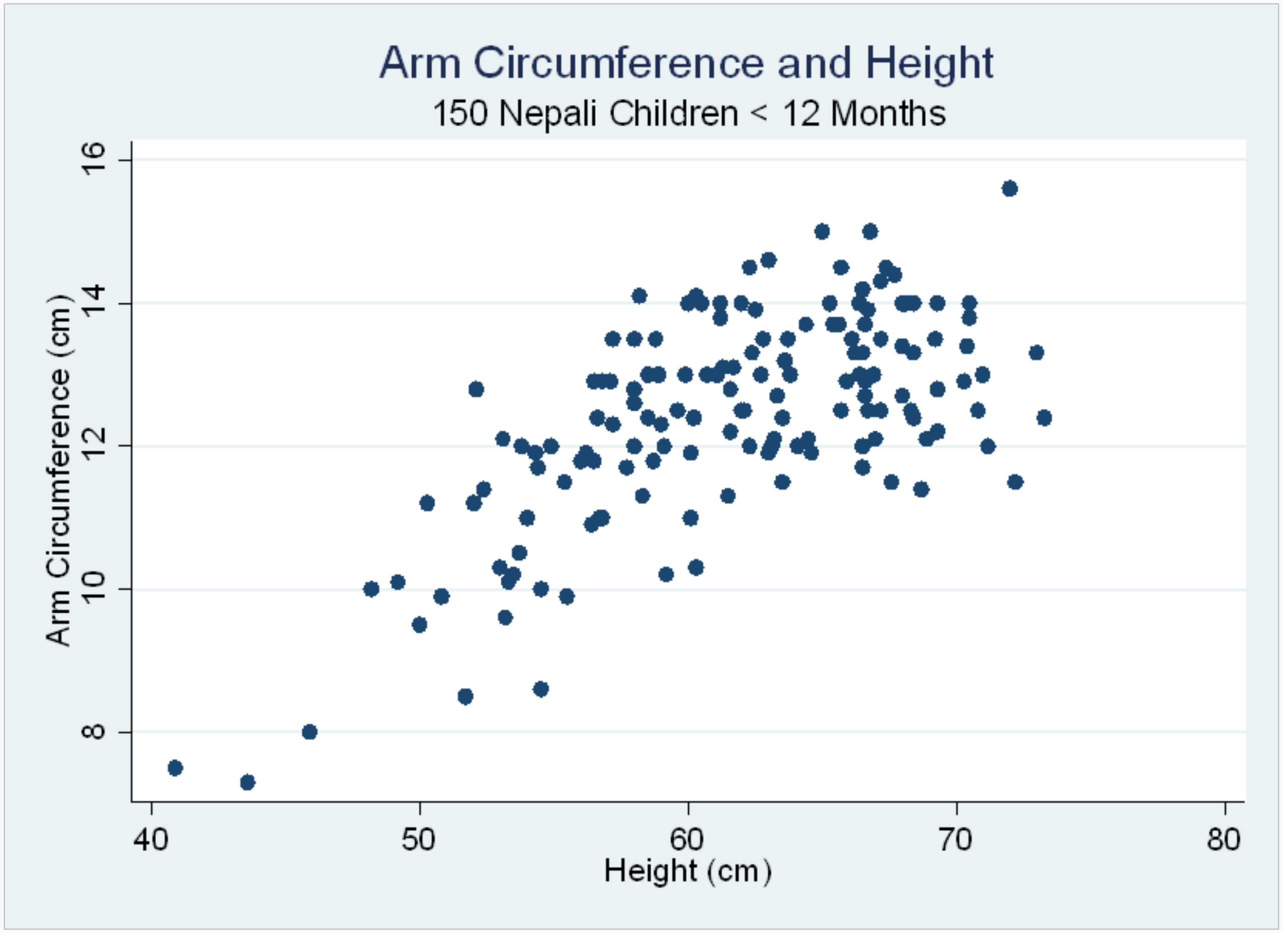
- What about treating height as continuous when estimating the arm circumference/height relationship

- Linear regression is a potential option: allows us to associate a continuous outcome with a continuous predictor via a line
  - The line estimates the mean value of the outcome for each continuous value of height in the sample used
  - Makes a lot of sense: but only if a line reasonably describes the outcome/predictor relationship

- Linear regression can also use binary or categorical predictors (will show later in this set of lectures)

Visualizing Arm Circumference and Height Relationship

- A useful visual display for assessing the nature of association between two continuous variables: a scatterplot



Visualizing Arm Circumference and Height Relationship

- Question: does a line reasonably describe the general shape of the relationship between arm circumference and height?

- We can estimate a line, using the computer (details to come in subsequent lecture section)

- The line we estimate will be of the form:

$$\hat{y} = \beta_o + \beta_1 x$$

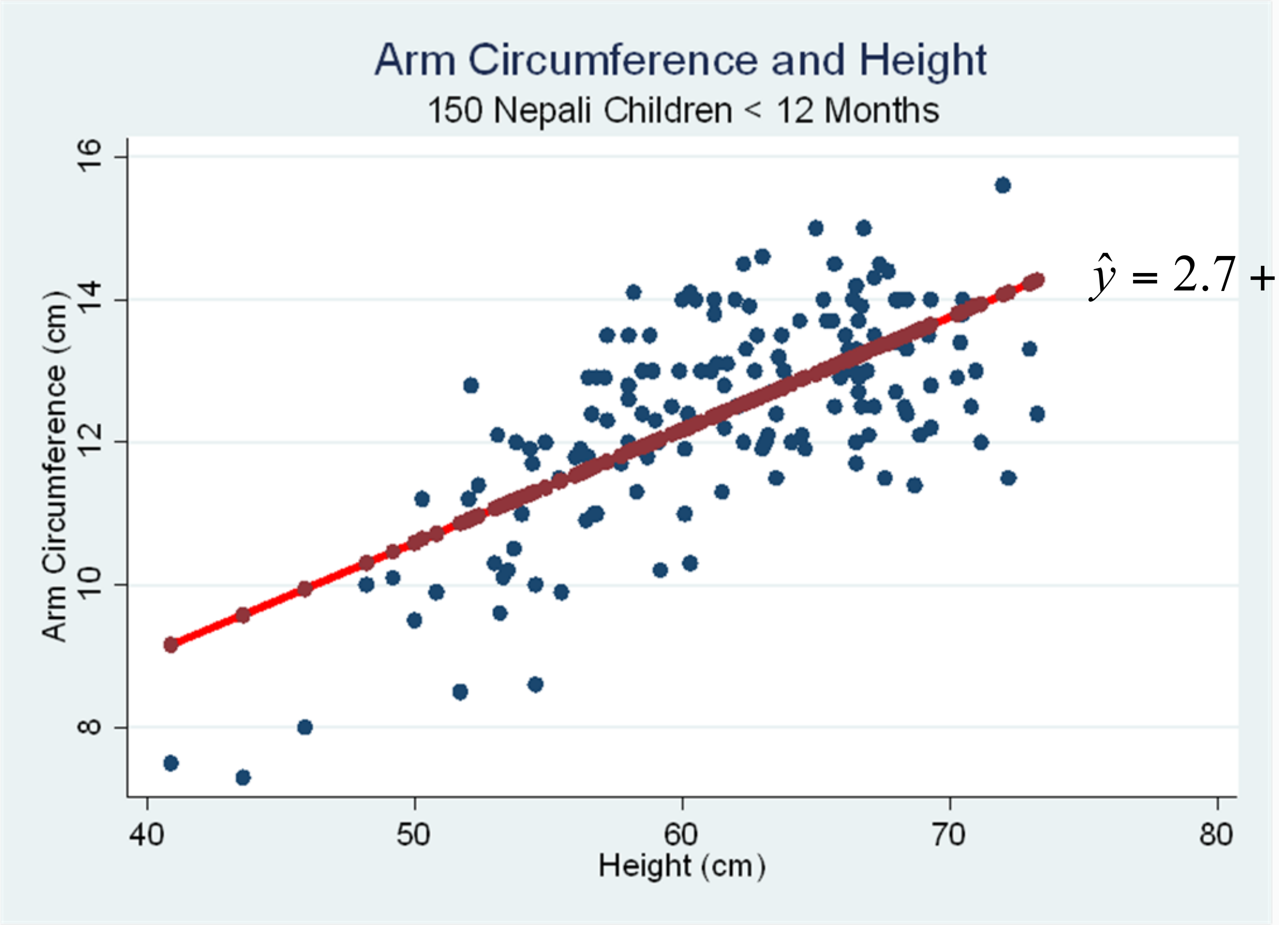
- Here $\hat{y}$ is the average arm circumference for a group of children all of the same height, x

Example: Arm Circumference and Height

- Equation of regression line relating estimated mean arm circumference (cm) to height (cm): from Stata
  - $\hat{y} = 2.7 + 0.16x$
  - Here, $\hat{y} =$ estimated average arm circumference (like what we previously would call $\overline{y}$), *x* = height, $\hat{\beta}_o = 2.7$ and $\hat{\beta}_1 = 0.16$
  - This is the estimated line from the sample of 150 Nepali children

Example: Arm Circumference and Height

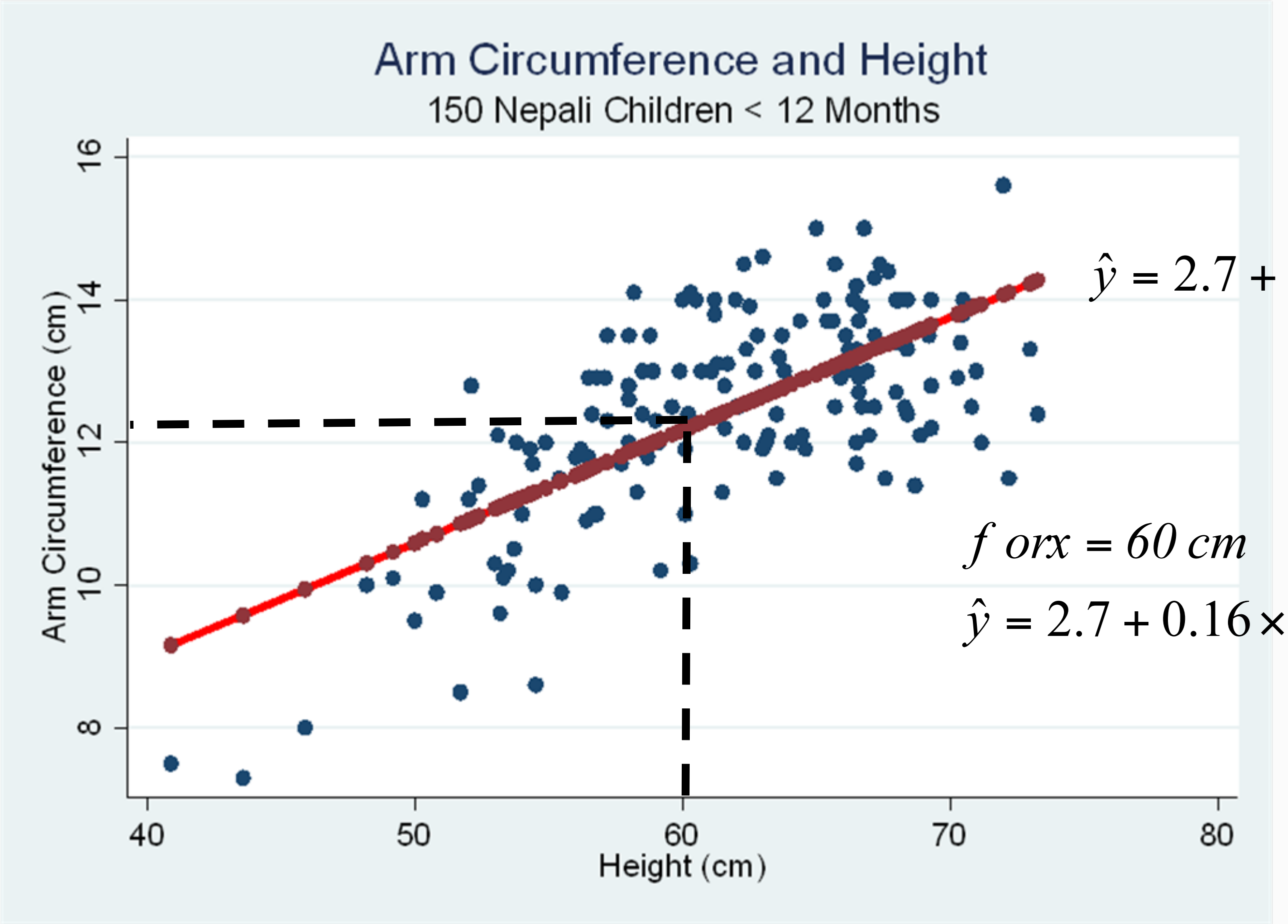
- Scatterplot with regression line superimposed



$\hat{y} = 2.7 + 0.16x$

Example: Arm Circumference and Height

- Estimated mean arm circumference for children 60 cm in height



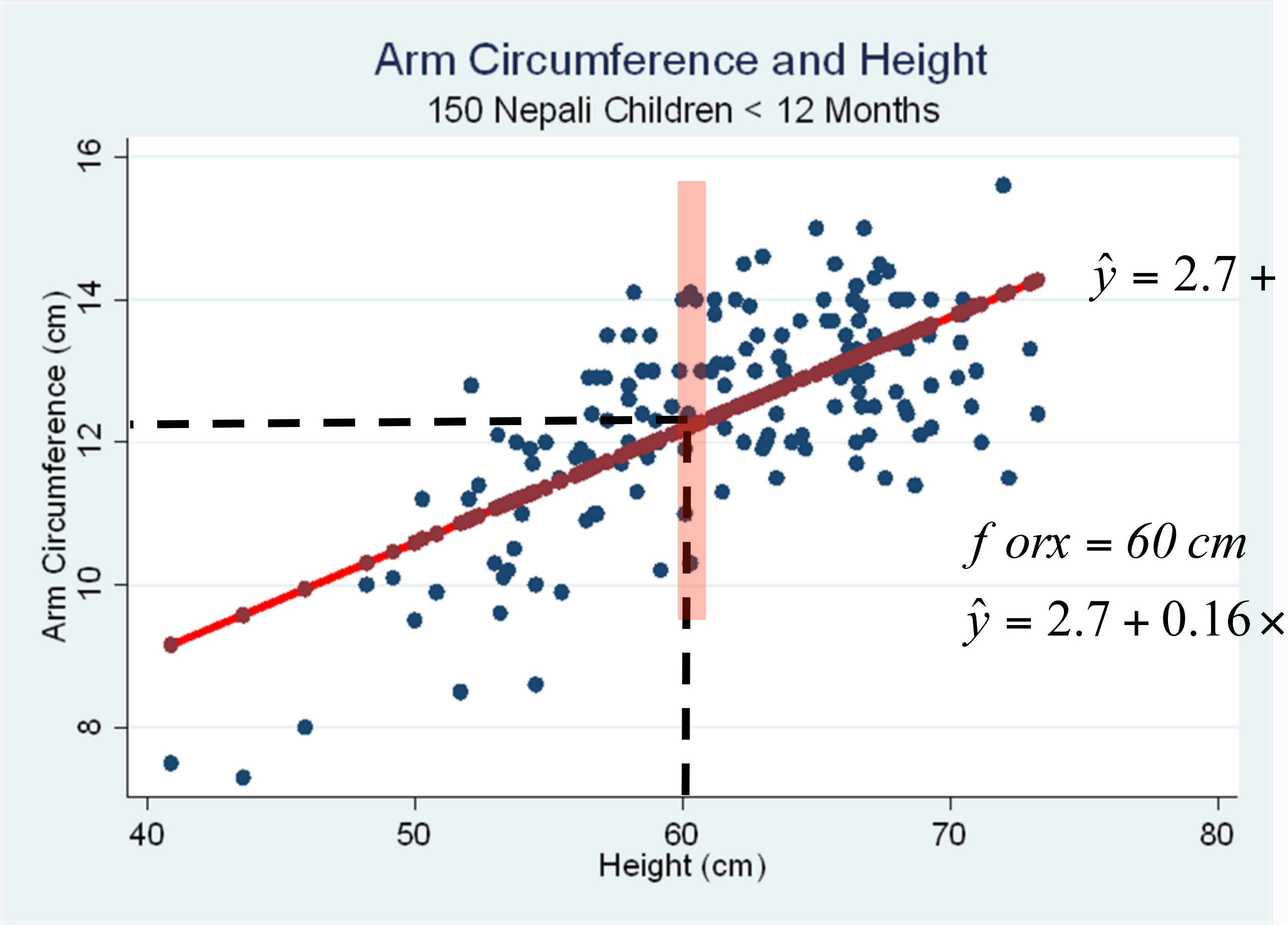
$$\hat{y} = 2.7 + 0.16x$$

$$f\ orx = 60\ cm$$

$$\hat{y} = 2.7 + 0.16 \times 60 = 12.3\ cm$$

Example: Arm Circumference and Height

- Notice, most points don't fall directly on the line: we are estimating the mean arm circumference of children 60 cm tall: observed points vary about the estimated mean



$$\hat{y} = 2.7 + 0.16x$$

$$f\ orx = 60\ cm$$

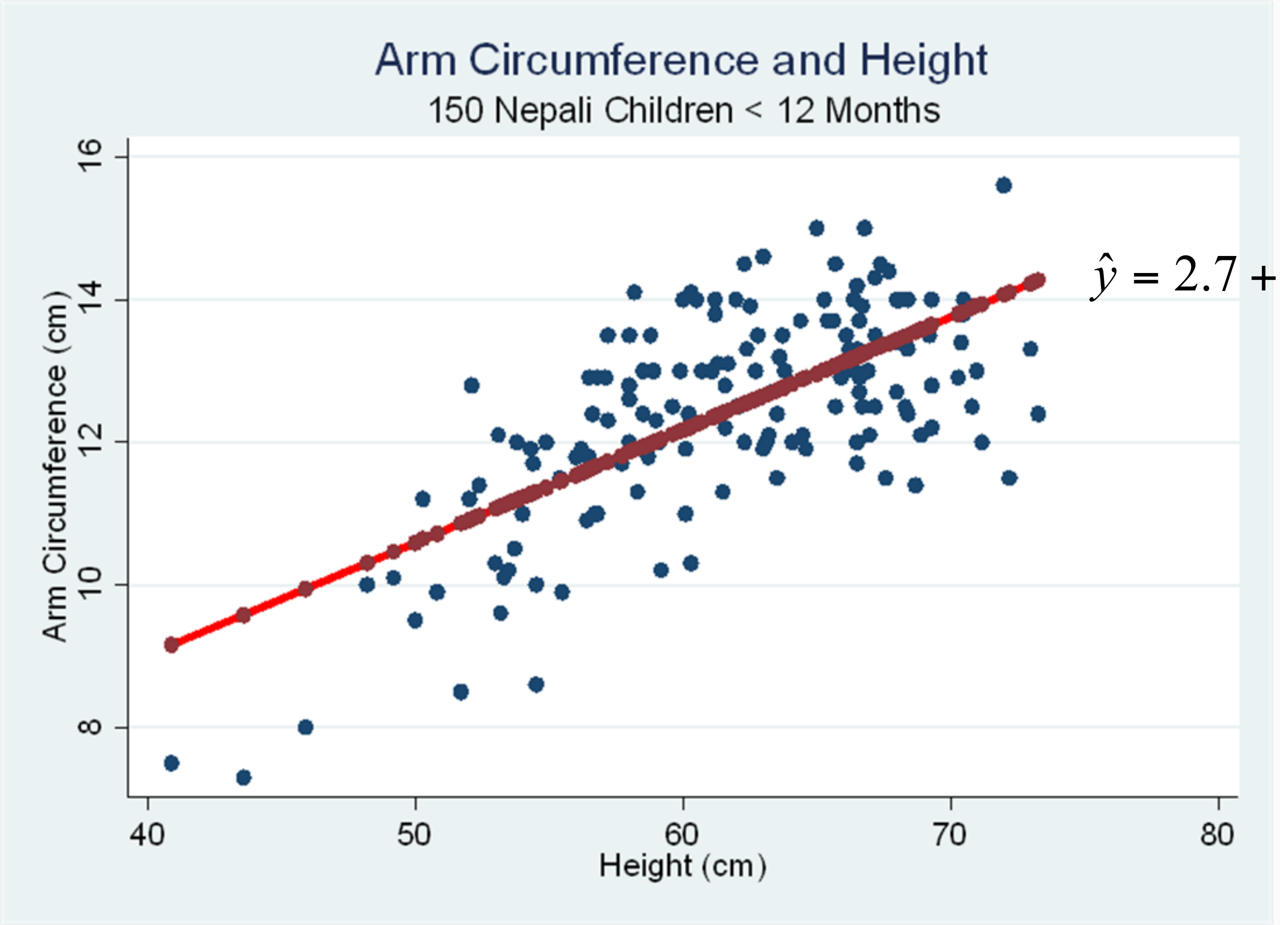
$$\hat{y} = 2.7 + 0.16 \times 60 = 12.3\ cm$$

Example: Arm Circumference and Height

- How to interpret estimated slope?
  - $\hat{y} = 2.7 + 0.16x$
  - Here, $\hat{\beta}_1 = 0.16$
  - Two ways to say the same thing:
    - $\hat{\beta}_1$ is the average change in arm circumference for a one-unit (1 cm) increase in height
    - $\hat{\beta}_1$ is the mean difference in arm circumference for two groups of children who differ by one-unit (1 cm) in height, taller to shorter
    - *These results estimate that the mean difference in arm circumferences for a one cm difference in height is 0.16 cm, with taller children having greater average arm circumference*

Example: Arm Circumference and Height

- This mean difference estimate is constant across the entire height range in the sample: definition of a slope of a line



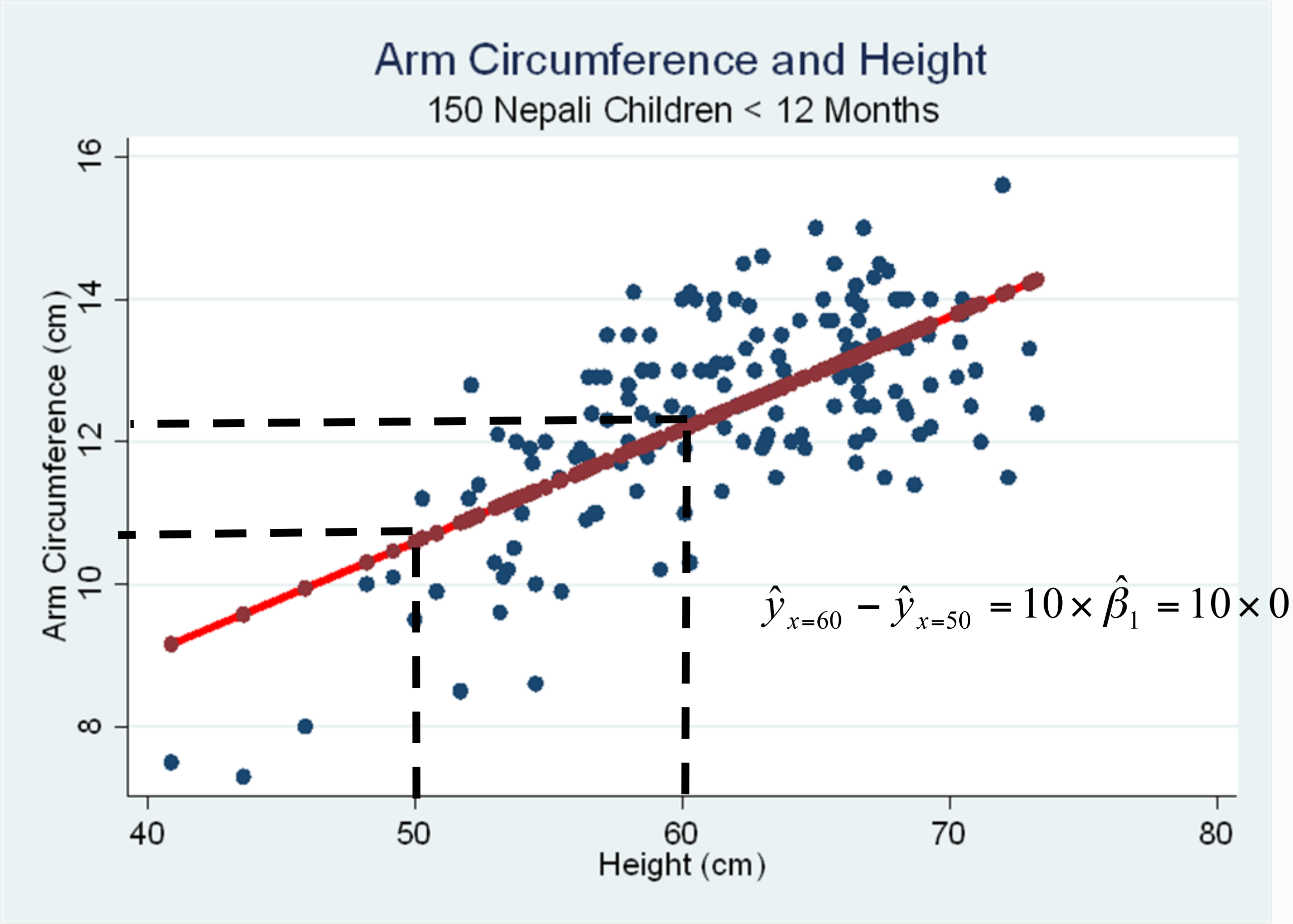
$\hat{y} = 2.7 + 0.16x$

Example: Arm Circumference and Height

- What is the estimated mean difference in arm circumference for:
  - Children 60 cm tall versus children 59 cm tall?
  - Children 25 cm tall versus children 24 cm tall?
  - Children 72 cm tall versus children 71 cm tall?
  - Etc.?
  - Answer is the same for all of the above: 0.16 cm

Example: Arm Circumference and Height

- What is estimated mean difference in arm circumference for . . .
  - Children 60 cm tall versus children 50 cm tall?



$$\hat{y}_{x=60} - \hat{y}_{x=50} = 10 \times \hat{\beta}_1 = 10 \times 0.16\,cm = 1.6\,cm$$

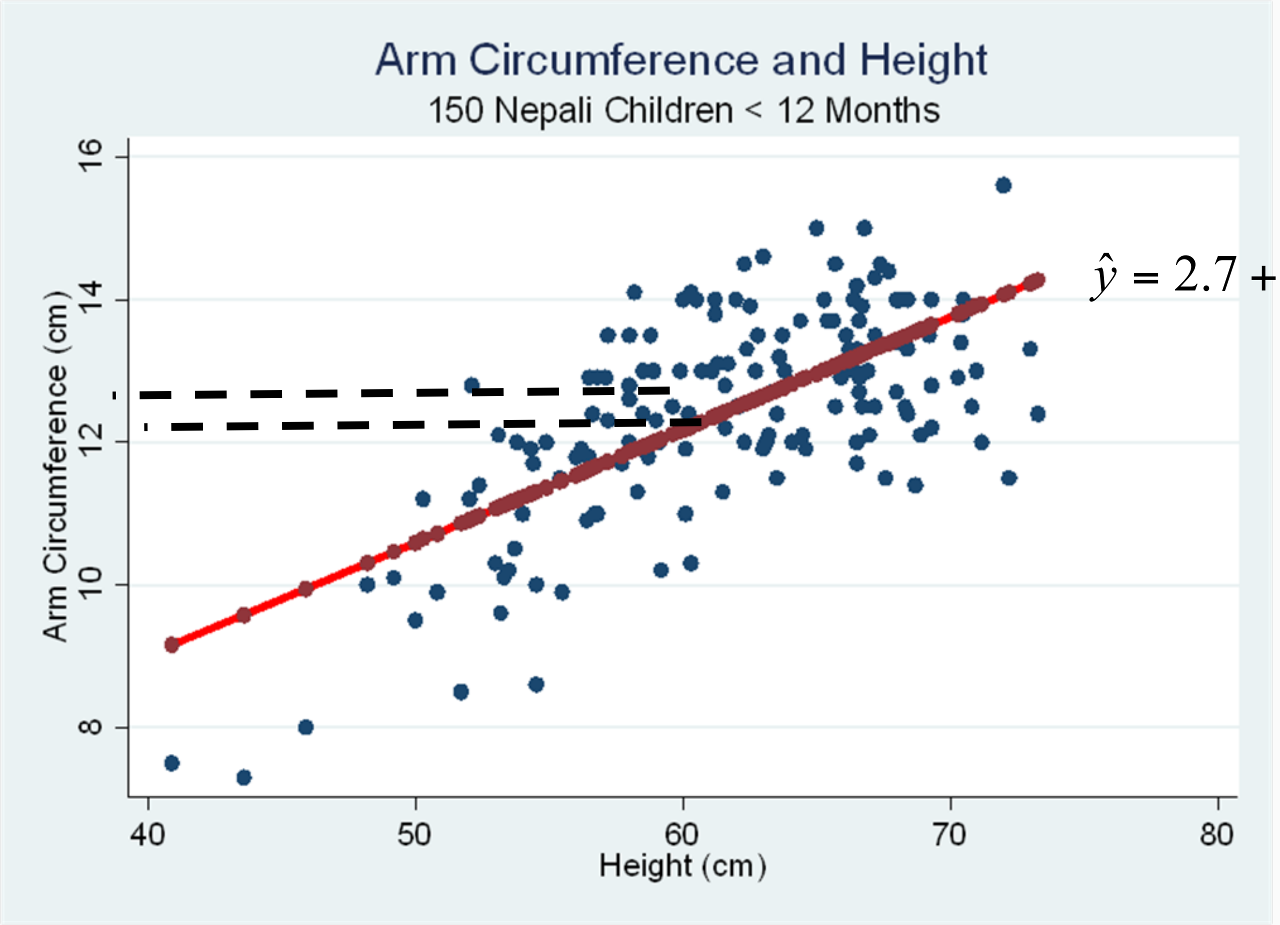
Example: Arm Circumference and Height

- What is estimated mean difference in arm circumference for . . .
  - Children 90 cm tall versus children 89 cm tall?
  - Children 34 cm tall versus children 33 cm tall?
  - Children 110 cm tall versus children 109 cm tall?
  - Etc.?

  - This is a trick question!

Example: Arm Circumference and Height

- The range of observed heights in the sample is 40.9 cm – 73.3 cm: our regression results only apply to the relationship between arm circumference and height for this height range



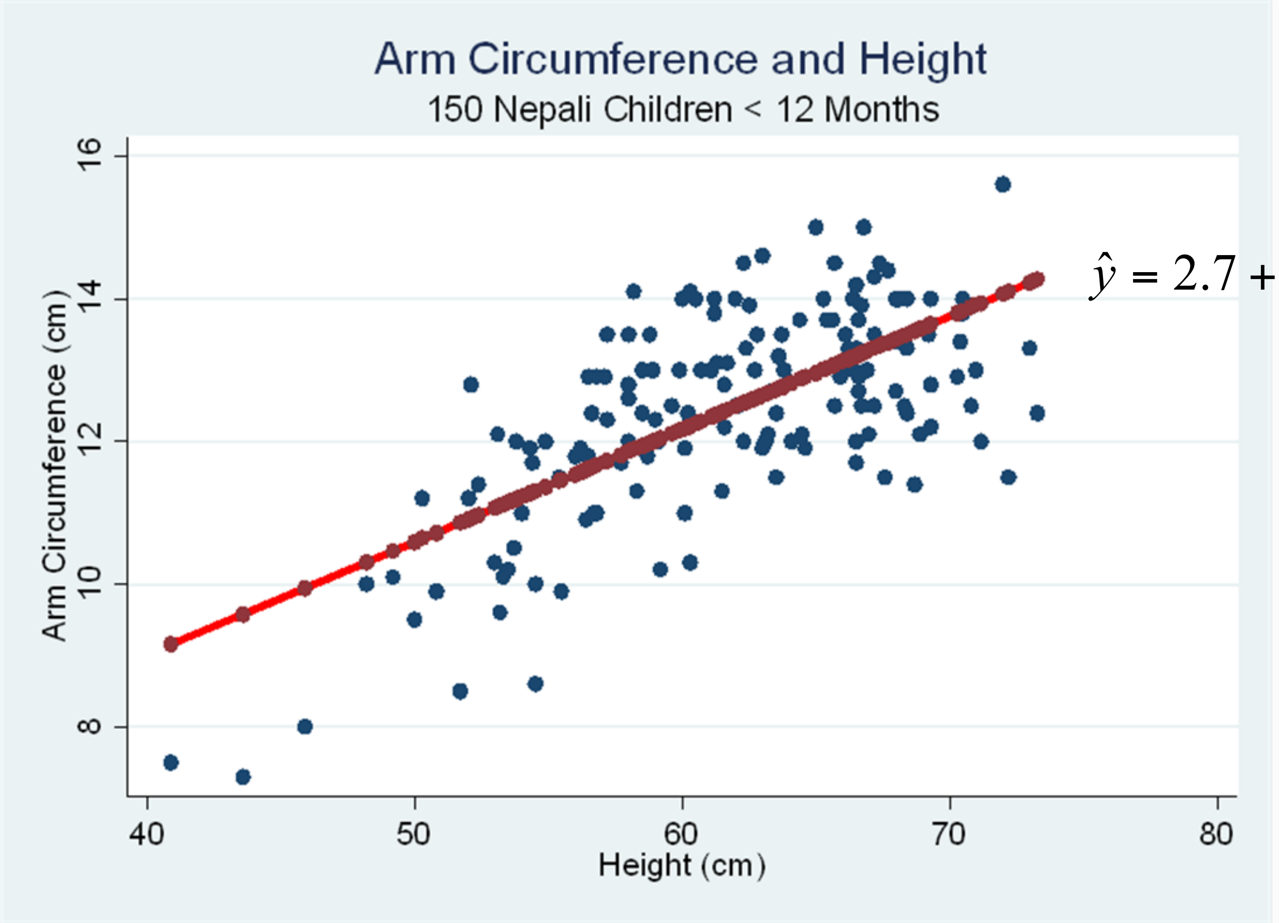
$$\hat{y} = 2.7 + 0.16x$$

Example: Arm Circumference and Height

- How to interpret estimated intercept?
  - $\hat{y} = 2.7 + 0.16x$
  - Here, $\hat{\beta}_o = 2.7cm$
  - This is the estimated y when x = 0: the estimated mean arm circumference for children 0 cm tall
    - Does this make sense given our sample?
    - As we noted before, the estimate of mean arm circumferences only applies to observed height range
    - Frequently, the scientific interpretation of the intercept is scientifically meaningless
    - But this intercept is necessary for fully specifying the equation of a line and making estimates of mean arm circumference for groups of children with heights in the sample range

Example: Arm Circumference and Height

- Notice that x = 0 is not even on this graph (the vertical axis is at x = 39)



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Example: Arm Circumference and Height

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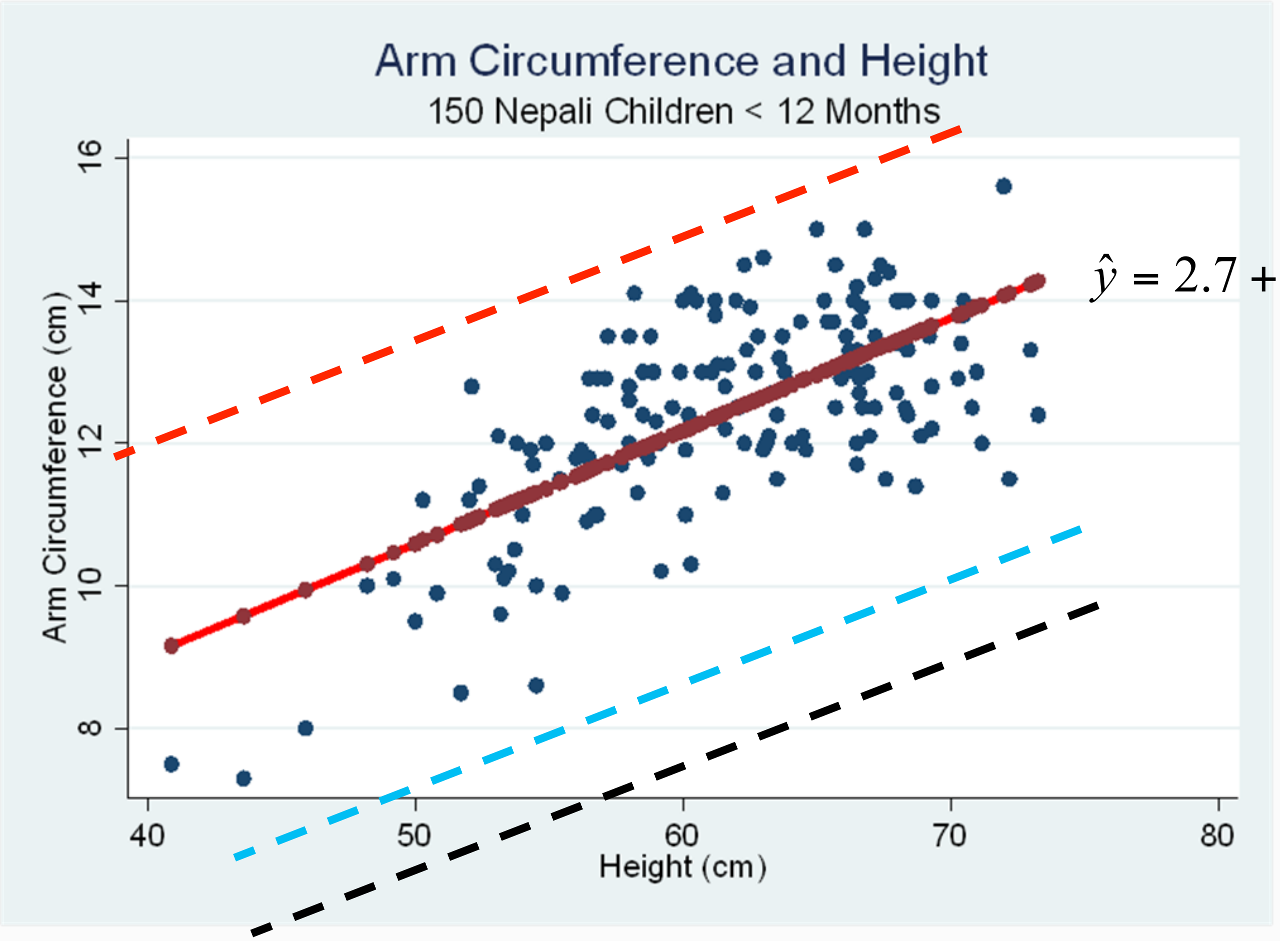

$\hat{y} = 2.7 + 0.16x$