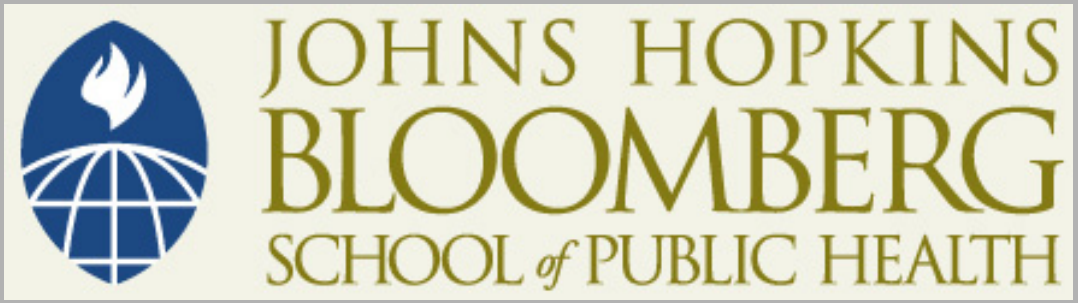


# Concluding Topics

Statistics for Psychosocial Research II:
Structural Models

Qian-Li Xue

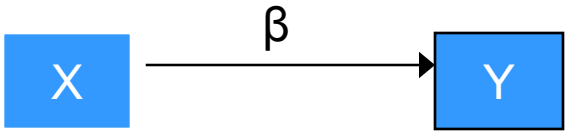
# Outline

- Brief discussion of Standardized Coefficients
- Causal inference
- Design, power, sample size

# Standardized Coefficients

- Make “relative” direct influences clearer
- Standardized coefficient of β:

$$\beta^* = \beta \frac{\sigma_x}{\sigma_y}$$




- The standardized effect is the mean change in standard deviation units of Y for a one standard deviation change in X.
- If X increases by one standard deviation, then we expect that Y will increase by β* standard deviations
- Standardized can be easier for making inferences from SEMs

# Quick Little Derivation

Model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$

Standardizing Variables

$$x_1^* = x_1 / \sigma_{x_1}; \quad x_2^* = x_2 / \sigma_{x_2}; \quad y^* = y / \sigma_y$$

$$\sigma_{x_1} x_1^* = x_1; \quad \sigma_{x_2} x_2^* = x_2; \quad \sigma_y y^* = y$$

Substitute standardized variables into model:

$$\sigma_y y^* = \beta_0 + \beta_1 \sigma_{x_1} x_1^* + \beta_2 \sigma_{x_2} x_2^* + e$$

$$y^* = \frac{\beta_0}{\sigma_y} + \beta_1 \frac{\sigma_{x_1}}{\sigma_y} x_1^* + \beta_2 \frac{\sigma_{x_2}}{\sigma_y} x_2^* + e$$

# Standardized Coefficients

- MPLUS provides different types of "standardized" estimates
  - StdYX – $\beta^* = \beta \frac{\sigma_x}{\sigma_y}$
  - StdY – $\beta^* = \beta / \sigma_y$
    - more appropriate for binary variable
    - interpreted as the change in standard deviation units of y when x changes from zero to one
  - Std – uses the variances of the continuous latent variables for standardization
  - StdYX and Std are the same for parameter estimates involving only latent variables

Mplus User's Guide

# Standardized Coefficients

- Caveats:
  - Beware of comparing standardized coefficients for the same variable across groups!
    - Standardized effects might be different due to different standard deviations
    - **Look at unstandardized coefficients when comparing across groups**

# Latent Variables in SEM

- Much like path analysis with observed variables
- Some additional considerations

# Constraining Latent Variables

- Recall: 2 options for CFA model
  - set variance of (exogenous) LV equal to 1 (my preference)
  - set path coefficient to one of its indicators equal to 1. This scales the LV in the same units as the indicator.
- More complicated models:
  - endogenous LV
    - set variance of its error term to 1 or
    - set path coefficient to one of its indicators to 1
- MUST do one of these or model is not identifiable!

# Causal Inference

# Many views on this topic!

- Correlation ≠ Causation
- But, coupled with other information, correlation can imply causation
- Statistics helps a lot with causal inference
- Statistical models used to draw causal inferences are distinctly different from those used for showing associational differences

# ‘Potential’ Cause

- Holland (1986): each ‘unit’ of observation must be able to be ‘exposed’ to the cause

- For causal inference, cause must be subject to “human manipulation.”

- Does
  - Smoking cause lung cancer?
  - Sleet or snow cause traffic accidents?
  - A change in interest rates cause the stock market to fluctuate?
  - Gender or race cause discrimination?

# “Potentially Exposable”

- Every ‘unit’ should be able to be exposed to the cause.
- Good example:  randomized clinical trial
- Need to be able to postulate that we could state what WOULD have happened to a patient’s outcome had the cause been “the reverse”.
  - Assume $Y_{ti}$ is the outcome of $Y_i$ if patient $Y_i$ is in the treatment group
  - Assume $Y_{ci}$ is the outcome of $Y_i$ if patient $Y_i$ is in the control group
- We are interested in the causal effect: $Y_{ti} - Y_{ci}$

# Fundamental Problem of Causal Inference

"It is impossible to *observe* the value of $Y_{ti}$ and $Y_{ci}$ on the same patient. Therefore it is impossible to observe the causal effect of treatment on patient $Y_i$."

- Important point: 'observe' is key word.
- Possible exceptions: cross-over designs in some settings.
- However, we CAN estimate 'average' causal effects over a population of patients
- Topic of the new course offered in 3rd and 4th term

# Causality

- Strong assumption of causality in SE models

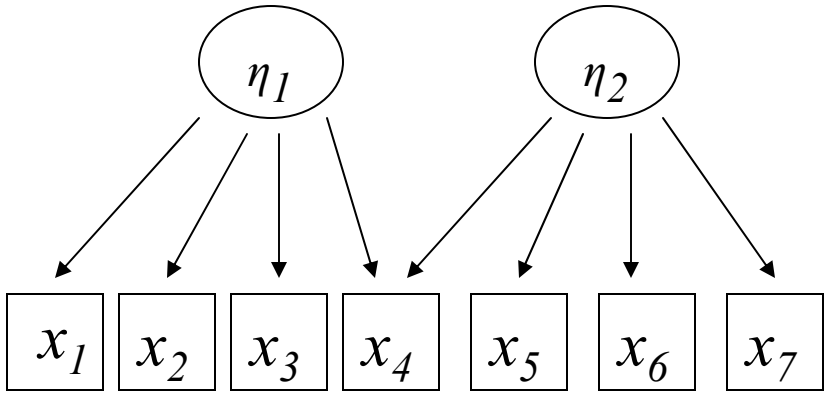
- Bollen's three components for 'practically' defining a causal relationship:
  - Association
  - Direction of influence
  - Isolation

# Association

- Easier to establish
- Causal variable should have strong association with outcome
- Problems:
  - Incorrect standard errors or test statistics (e.g. correlated errors, poor measures)
  - Multicollinearity
- Replication/Repetition important (and also helps establish isolation)

# Multicollinearity Example

$\eta_1$: morale; $\eta_2$: sense of belonging




$$x_1 = \gamma_{11}\eta_1 + \zeta_1$$
$$\vdots$$
$$x_4 = \gamma_{14}\eta_1 + \gamma_{24}\eta_2 + \zeta_4$$
$$\vdots$$
$$x_7 = \gamma_{27}\eta_2 + \zeta_7$$

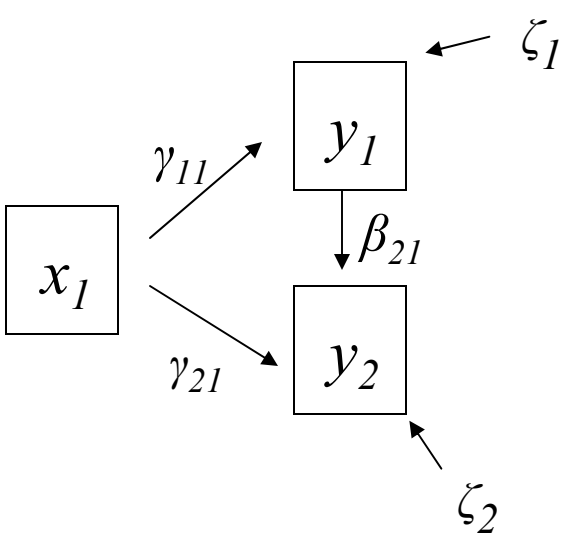
- In truth, *x4* is a measure of morale, but we allow it to be related to sense of belonging
- Results? Both *γ14* and *γ24* are insignificant
- Why? Because morale and sense of belonging are highly associated

# Direction of Causation

- Plausibility of association being causal rests on having causal direction correct
- Temporal?
  - x should come before y in time
  - problematic: simultaneous reciprocal causation (feedback) is not possible
  - window of cause and response time
- We often have cross-sectional data.
- Can future event predict past or present event?

# Recap: Total, Direct, and Indirect Effects

- $x_1$ is marital status, $y_1$ is income, $y_2$ is depression
- Direct effect: measured by a single arrow between two variables
- Indirect effects: measured by all possible "paths" or "connections" between two variables EXCEPT for the direct path. We multiply the coefficients on path together to get each indirect effect.
- Total effect: the sum of the direct and indirect paths between two variables




Direct effect of $x_1$ on $y_2$:

Indirect effect(s) of $x_1$ on $y_2$:

Total effect of $x_1$ on $y_2$:

---

Direct effect of $y_1$ on $y_2$:

Indirect effect(s) of $y_1$ on $y_2$:

Total effect of $y_1$ on $y_2$:

# Isolation

- Isolation:  hold everything constant except the cause and the outcome
- Impossible to establish unless x and y occur in a “vacuum”
- Especially difficult in observational studies!
- Without true isolation can never be 100% certain about cause
- Isolation tends to be the weakest link in determining causality!

# "Pseudo-isolation"

$$y_1 = \gamma_{11} x_1 + \zeta_1$$

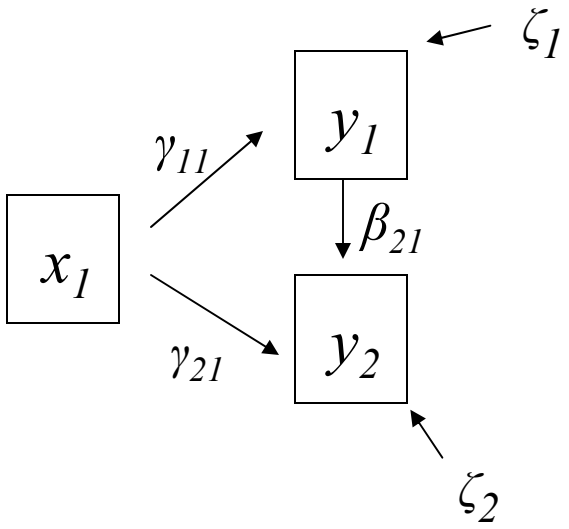
$\zeta_1$ is the unobserved error/disturbance

$\zeta_1$ represents ALL other causes/correlates of $y_1$

- Standard assumption for pseudo-isolation: $Cov(x_1, \zeta_1) = 0$
- That is, $x_1$ is independent of all other causes/correlates of $y_1$
- If the assumption is true, then we can assess causal association of $x_1$ and $y_1$ "isolated" from all other causes ($\zeta_1$).

# Examples of Violations of Isolation

## (1) Intervening Variables




True Model:

$$y_1 = \gamma_{11} x_1 + \zeta_1$$

$$y_2 = \beta_{21} y_1 + \gamma_{21} x_1 + \zeta_2$$

$$Cov(\zeta_1, \zeta_2) = 0$$

$$Cov(x_1, \zeta_1) = 0$$

$$Cov(x_1, \zeta_2) = 0$$

(e.g. $x_1$ is marital status, $y_1$ is household income, $y_2$ is depression)

(Bollen, p.46)

# What if we omit $y_1$ (income)?

- Assumed model:

$$y_2 = \gamma^*_{21} x_1 + \zeta^*_2$$

- This implies:

$$\zeta^*_2 = \beta_{21} y_1 + \zeta_2$$

- And our pseudo-isolation assumption….

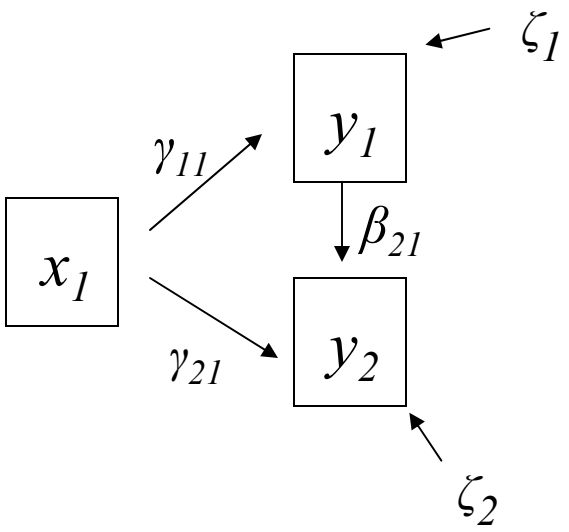
$$\begin{aligned} Cov(x_1, \zeta^*_2) &= Cov(x_1, \beta_{21} y_1 + \zeta_2) \\ &= Cov(x_1, \beta_{21}(\gamma_{11} x_1 + \zeta_1) + \zeta_2) \\ &= \beta_{21}\gamma_{11} Var(x_1) + \beta_{21} Cov(x_1, \zeta_1) + Cov(x_1, \zeta_2) \\ &= \beta_{21}\gamma_{11} Var(x_1) \\ &\neq 0 \end{aligned}$$

# Effect on Inference?

- $\gamma_{21}^{*}$ converges to total effect, $\beta_{21}\gamma_{11} + \gamma_{21}$, instead of direct effect, $\gamma_{21}$
- This yields an over or under-estimate of the effect of $x_1$ on $y_2$.
- Can be a really big problem if direct and indirect effects cancel each other out.
  - If $\gamma_{21} = 1$; $\beta_{21} = 0.5$; $\gamma_{11} = -2$
  - Then, $\gamma_{21}^{*} = -0.5*2 + 1 = 0$
  - We might conclude that there is NO association!

# Examples of Violations of Isolation

## (2) Left Out Common Cause




Recall True Model:

$$y_1 = \gamma_{11} x_1 + \zeta_1$$

$$y_2 = \beta_{21} y_1 + \gamma_{21} x_1 + \zeta_2$$

$$Cov(\zeta_1, \zeta_2) = 0$$

$$Cov(x_1, \zeta_1) = 0$$

$$Cov(x_1, \zeta_2) = 0$$

What if we omit $x_1$ from the model?

$$\text{Then } y_2 = \beta^*_{21} y_1 + \zeta^*_2$$

$$\text{where } \zeta^*_2 = \gamma_{21} x_1 + \zeta_1$$

# Examples of Violations of Isolation

- Is pseudo-isolation assumption violated?

$$\begin{aligned} Cov(y_1, \zeta_2^*) &= Cov(\gamma_{11} x_1 + \zeta_1, \gamma_{21} x_1 + \zeta_2) \\ &= \gamma_{11}\gamma_{21} Var(x_1) \\ &\neq 0 \end{aligned}$$

- What happens to our estimate of $\beta_{21}$ ?

$$\beta_{21}^* = \beta_{21} + \gamma_{21} b_{11}$$

Where $b_{11}$ is the regression coefficient from an "auxiliary" regression of $x_1$ on $y_1$

# Effects on Inference

- Worst case scenario: $y_1$ and $y_2$ have little or no association, but both are highly associated with $x_1$.
- Example:

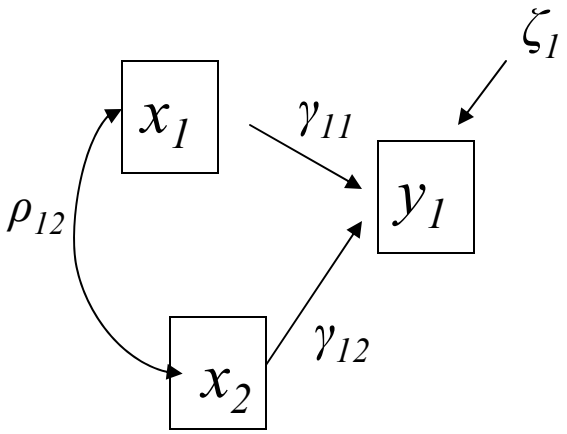
  $x_1$ = age

  $y_1$ = proportion of gray hairs

  $y_2$ = quality of vision
- "Spurious Association"

# Examples of Violations of Isolation

(3) Omitted Variable Has Unspecified Relation To Other Variables




True Model:

$$y_1 = \gamma_{11} x_1 + \gamma_{12} x_2 + \zeta_1$$

What if we omit $x_2$?

$$y_1 = \gamma^*_{11} x_1 + \zeta^*_1$$

$$\gamma^*_{11} = \gamma_{11} + \rho_{12}\gamma_{12}$$

(Bollen, p.54)

# This is an even bigger problem…

- Note that the association between $x_1$ and $x_2$ is unspecified: It could be true that
  - $x_1$ causes $x_2$ and $y_1$ (intervening variable)
  - $x_2$ causes $x_1$ and $y_1$ (common cause)
  - something else
- We can't infer about the exact consequences of omitting $x_2$ because we don't know its association to the other variables.

# Other Violations

- "feedback" or "reciprocal causation"
- Wrong functional form between 2 variables
- Correlated errors
- Bottom line:
  - SEM CAN NOT be used to PROVE causation!
  - Instead, SEM can be used to
    - "provide estimates of the strength of all the hypothesized relationships between variables in a theoretical model"
    - Assess consistency between data and model
    - "Distinguish between or among alternative perspectives"

# Sample Size and Power

- Factor analysis
  - General rule: need larger sample size (~300-500) for lower communalities, more factors, and fewer indicators per factor (MacCallum, 1999)
- SEM
  - > 50 + 8 × number of variables in the model
  - 10 to 20 ×number of variables (Mitchell, 1993)
  - >15 cases per measured variable or indicator (Stevens,1996)
  - >5 cases per parameter estimate (including error terms as well as path coefficients) is model assumptions are met (Bentler and Chou, 1987)
  - Larger N when data are non-normal or incomplete
- Monte Carlo Simulation studies

# Monte Carlo Simualtion Study for a CFA

```
TITLE:    this is an example of a Monte Carlo
    simulation study for a CFA with covariates
    (MIMIC) with continuous factor indicators
    and patterns of missing data
MONTECARLO:
    NAMES ARE y1-y4 x1 x2;
    NOBSERVATIONS = 500;
    NREPS = 500;
    SEED = 4533;
    !GENERATE=
    CUTPOINTS = x2(1);
    PATMISS = y1(.1) y2(.2) y3(.3) y4(1) |
              y1(1) y2(.1) y3(.2) y4(.3);
    PATPROBS = .4 | .6;
MODEL POPULATION:
    [x1-x2@0];
    x1-x2@1;
    f BY y1@1 y2-y4*1;
    f*.5;
    y1-y4*.5;
    f ON x1*1 x2*.3;
MODEL:
    f BY y1@1 y2-y4*1;
    f*.5;
    y1-y4*.5;
    f ON x1*1 x2*.3;
OUTPUT:   TECH9;
```

Simulation of a hypothetical population

Numer of samples drawn

generate different types of dependent variables (default: continuous)

Generate binary covariate based on Normal(0,1) using threshold=1

Specify patterns of missing data, here we have two missing patterns

Provide true population parameter values for data simulation

MPLUS User's Guide (Ex.11.1)

# Monte Carlo Study Output

MODEL RESULTS

| | | ESTIMATES | | S. E. | M. S. E. | 95% | % Sig |
|---|---|---|---|---|---|---|---|
| | Population | Average | Std. Dev. | Average | | Cover | Coeff |
| F BY | | | | | | | |
| Y1 | 1.000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 1.000 | 0.000 |
| Y2 | 1.000 | 1.0028 | 0.0663 | 0.0622 | 0.0044 | 0.940 | 1.000 |
| Y3 | 1.000 | 1.0005 | 0.0674 | 0.0633 | 0.0045 | 0.930 | 1.000 |
| Y4 | 1.000 | 1.0020 | 0.0806 | 0.0743 | 0.0065 | 0.932 | 1.000 |
| F ON | | | | | | | |
| X1 | 1.000 | 1.0023 | 0.0671 | 0.0620 | 0.0045 | 0.938 | 1.000 |
| X2 | 0.300 | 0.2991 | 0.1065 | 0.1052 | 0.0113 | 0.958 | 0.810 |
| Intercepts | | | | | | | |
| Y1 | 0.000 | -0.0018 | 0.0718 | 0.0688 | 0.0052 | 0.950 | 0.050 |
| Y2 | 0.000 | 0.0022 | 0.0515 | 0.0500 | 0.0027 | 0.946 | 0.054 |
| Y3 | 0.000 | 0.0015 | 0.0535 | 0.0519 | 0.0029 | 0.944 | 0.056 |
| Y4 | 0.000 | 0.0005 | 0.0603 | 0.0644 | 0.0036 | 0.962 | 0.038 |
| Residual Variances | | | | | | | |
| Y1 | 0.500 | 0.4887 | 0.0776 | 0.0779 | 0.0061 | 0.944 | 1.000 |
| Y2 | 0.500 | 0.4982 | 0.0502 | 0.0513 | 0.0025 | 0.950 | 1.000 |
| Y3 | 0.500 | 0.4966 | 0.0558 | 0.0532 | 0.0031 | 0.926 | 1.000 |
| Y4 | 0.500 | 0.4905 | 0.0755 | 0.0697 | 0.0058 | 0.920 | 1.000 |
| F | 0.500 | 0.5019 | 0.0715 | 0.0679 | 0.0051 | 0.930 | 1.000 |